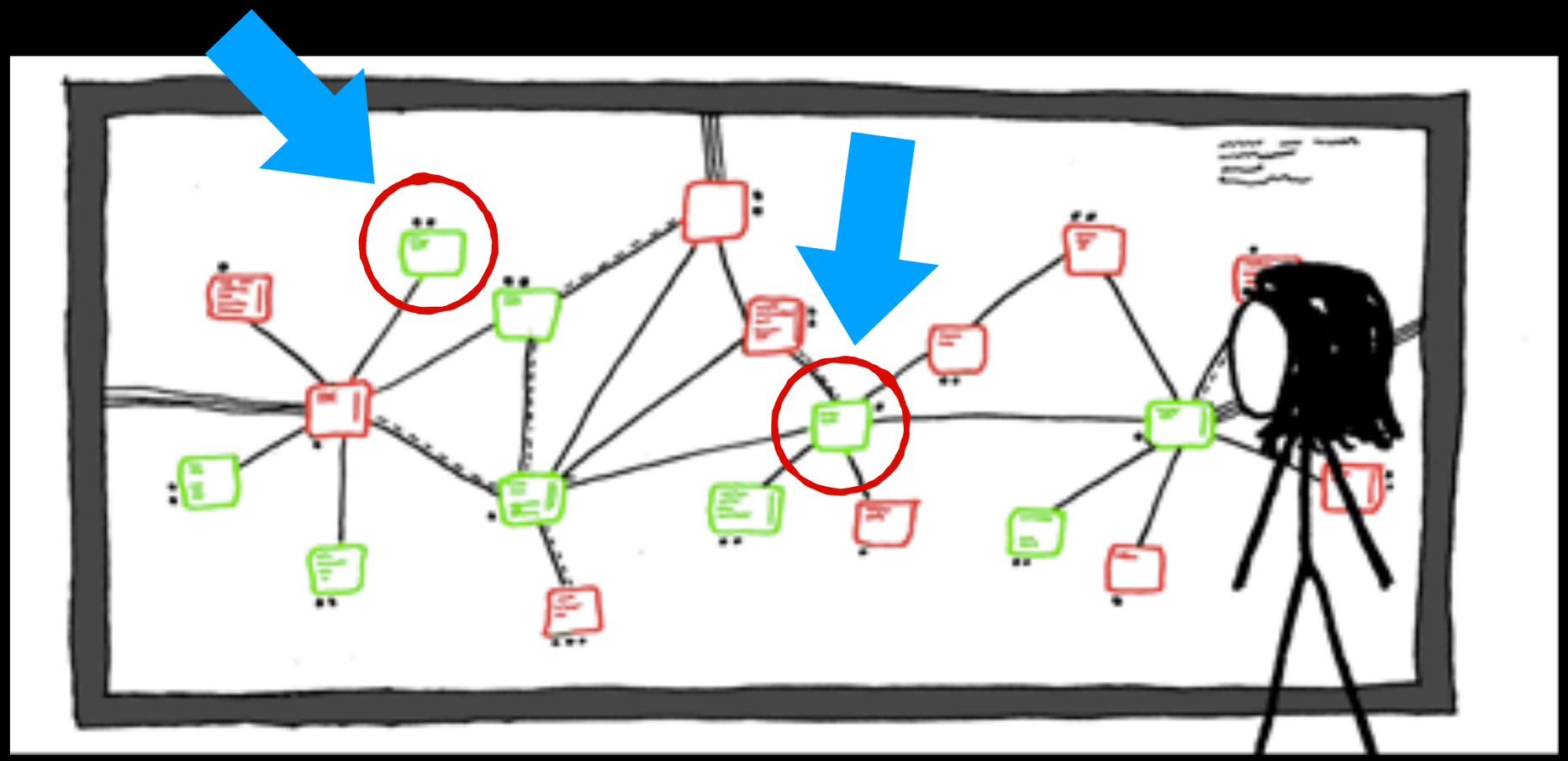
Properties of Networks

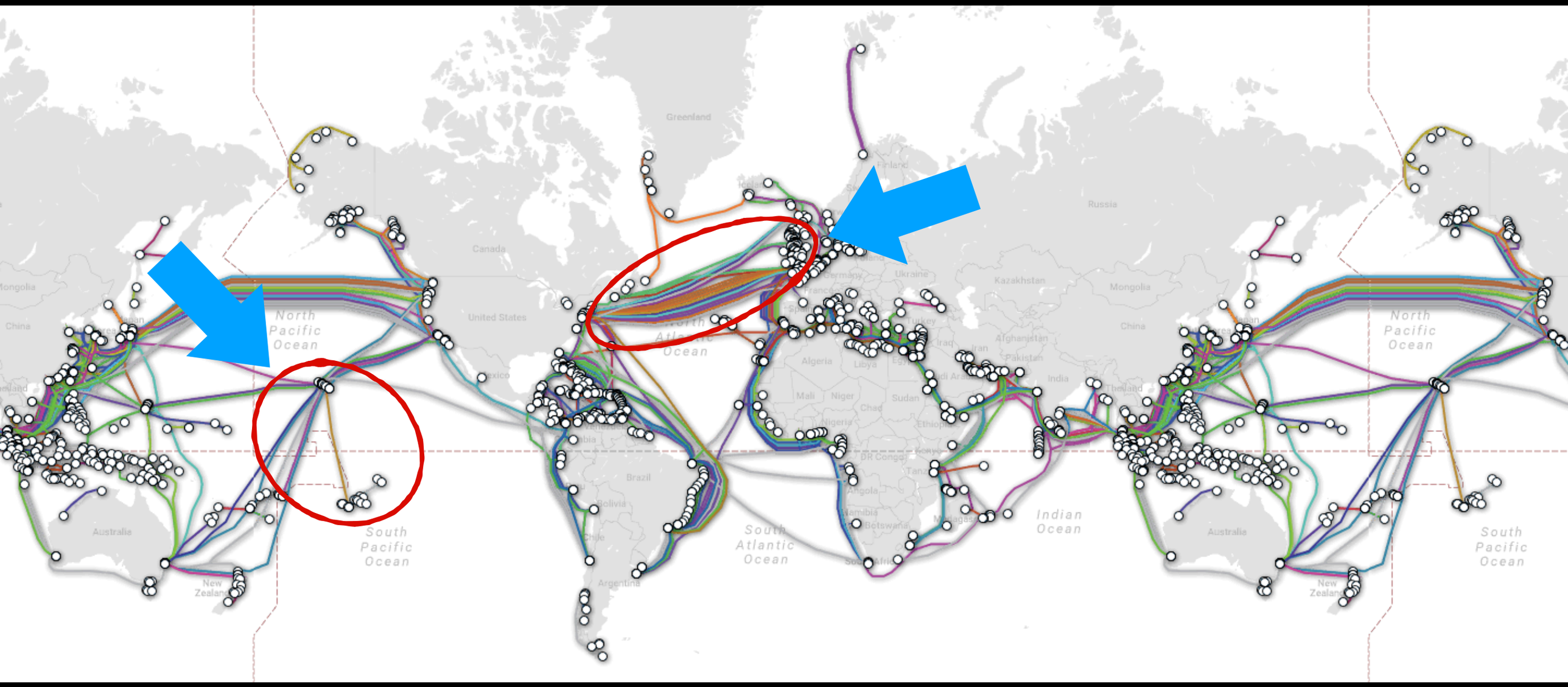
ENPM809G - Network Data Science

Overview

How Important Am I?

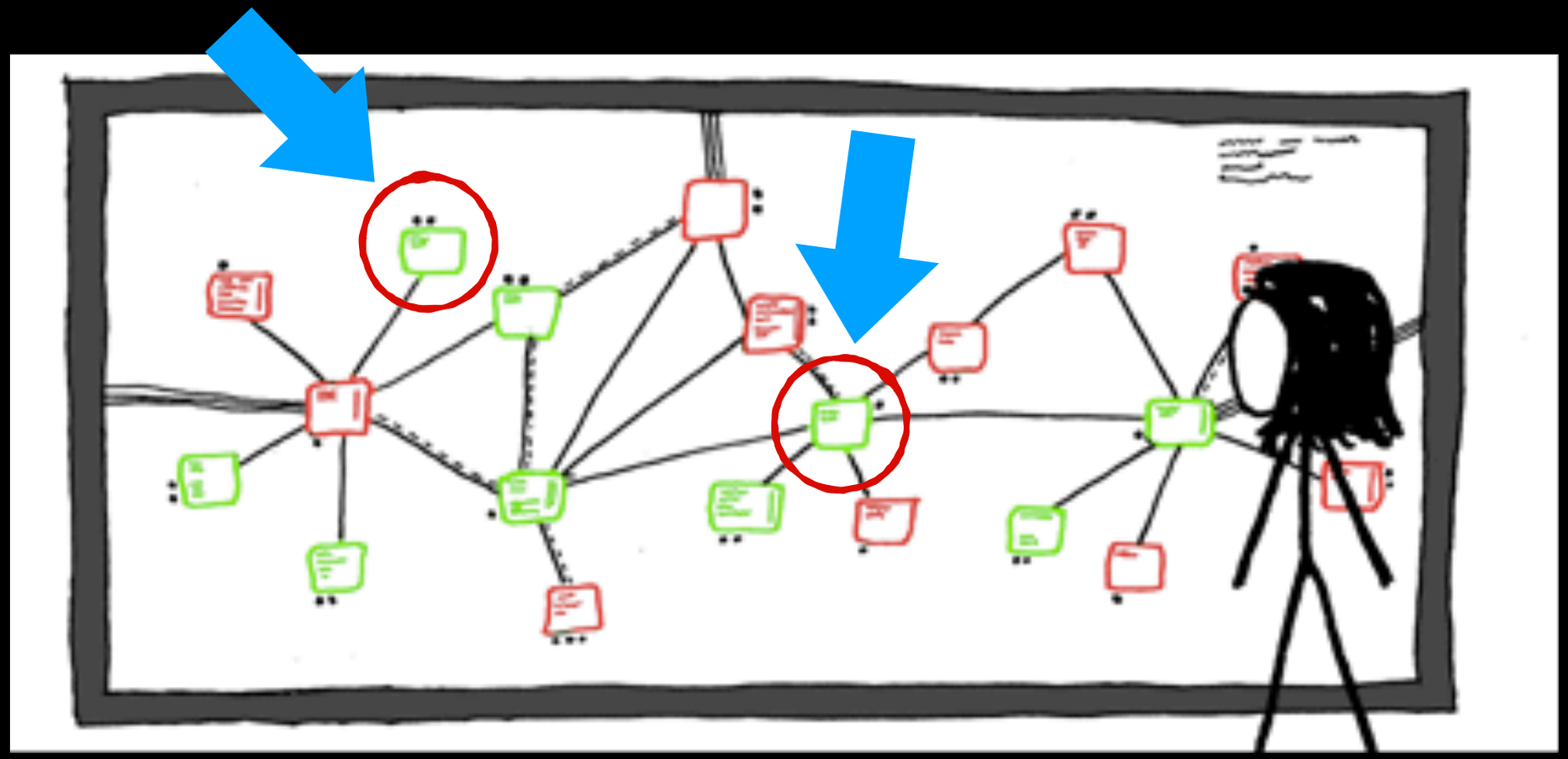


How Connected Are We?



Questions about HW1?

How Important Am I?



Additional Vocabulary

Types of Edges

- Unweighted Edge
- Weighted Edge
- Parallel Edges
  - “MultiGraphs”

Twinbrook
White Flint
Grosvenor-Strathmore
Medical Center
Bethesda
Friendship Heights
Tenleytown-AU
Van Ness-UDC
Cleveland Park
Woodley Park
Zoo/Adams Morgan
Dupont Circle
Farragut North
Georgia Ave-Petworth
Takoma
YL
GR
Greenbelt
College Park-U of Md
Prince George's Plaza
West Hyattsville
Fort Totten
Columbia Heights
U St
African-Amer Civil War Mem'l/Cardozo
Shaw-Howard U
Mt Vernon Sq
7th St-Convention Center
Gallery Place
Chinatown
Brookland-CUA
Rhode Island Ave
Brentwood
NoMa-Gallaudet U
Union Station
SV
Wiehle-Reston East
Spring Hill
Greensboro
Tysons Corner
McLean
McPherson Sq
Rosslyn
Foggy Bottom-GWU
Farragut West
Metro Center
Federal Triangle
Smithsonian
Archives
Navy Memorial-Penn Quarter
Judiciary Sq
National Mall
OR
New Carrollton
Landover
Cheverly
Deanwood
Minnesota Ave
Vienna
Dunn Loring
West Falls Church
East Falls Church
Ballston-MU
Virginia Sq-GMU
Clarendon
Court House
Arlington Cemetery
L'Enfant Plaza
Federal Center SW
Capitol South
Eastern Market
Potomac Ave
Stadium-Armory
Benning Rd
Capitol Heights
Addison Rd
Morgan Blvd
Largo Town Center
BL
Pentagon
Pentagon City
Waterfront
Navy Yard-Ballpark
Anacostia
Congress Heights
Southern Ave
Naylor Rd
Suitland
Branch Ave
Crystal City
Ronald Reagan Washington National Airport
Braddock Rd
King St-Old Town
Van Dorn St
Eisenhower Ave
Metrorail Operating Times
Mon-Thu
5am-midnight
Fri
5am-3am
Sat
7am-3am
Sun

Bipartite Networks

- Exception to the rule:
  - “Nodes/edges should mean the same thing”
- Two layers of nodes

Bipartite Networks

Bipartite Networks

- Other bipartite graphs?

Different Interpretations of Graphs

- Network Dual
- And bipartite networks

Network Dual

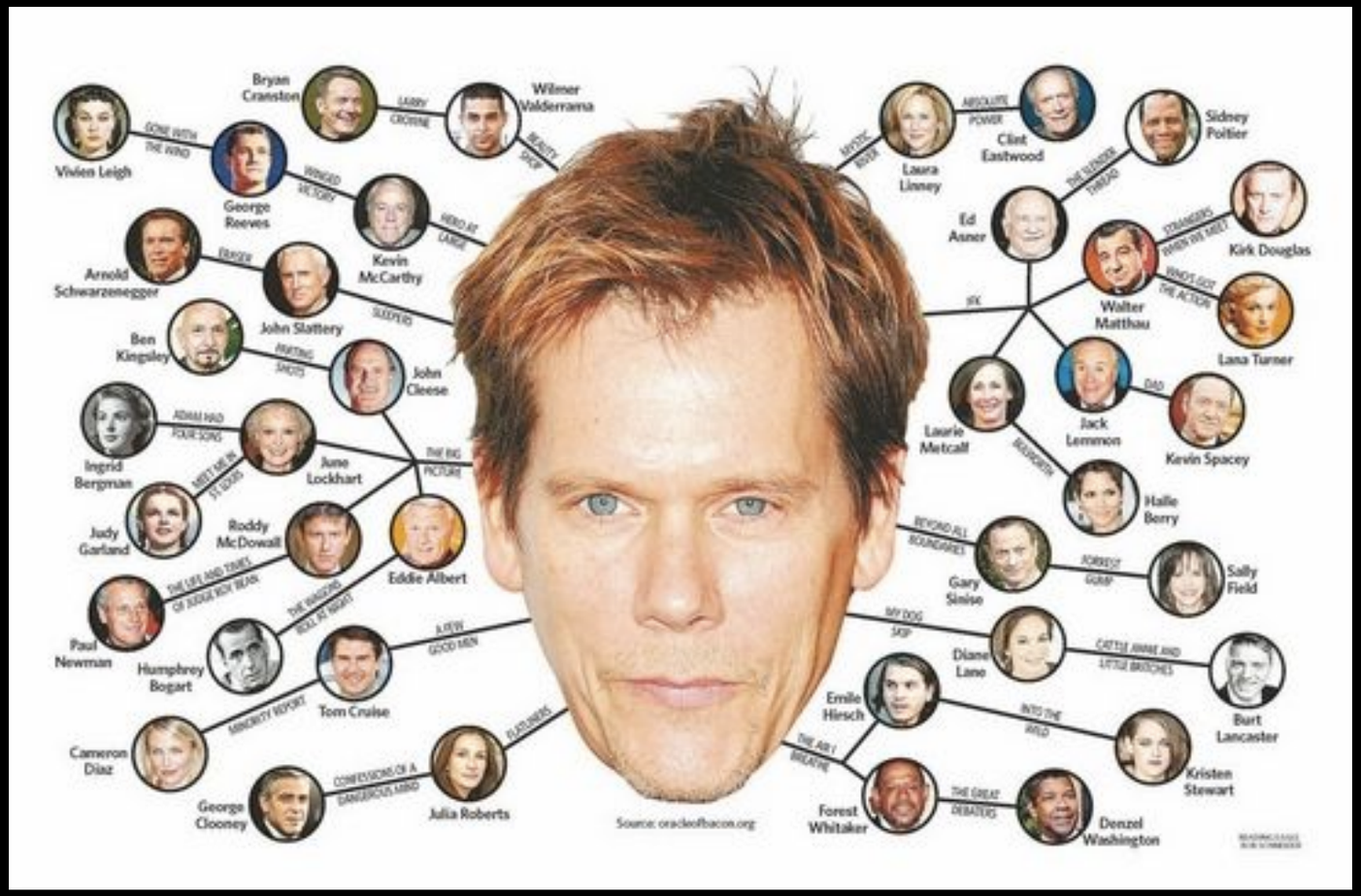
Properties of Nodes

What is “Degree”

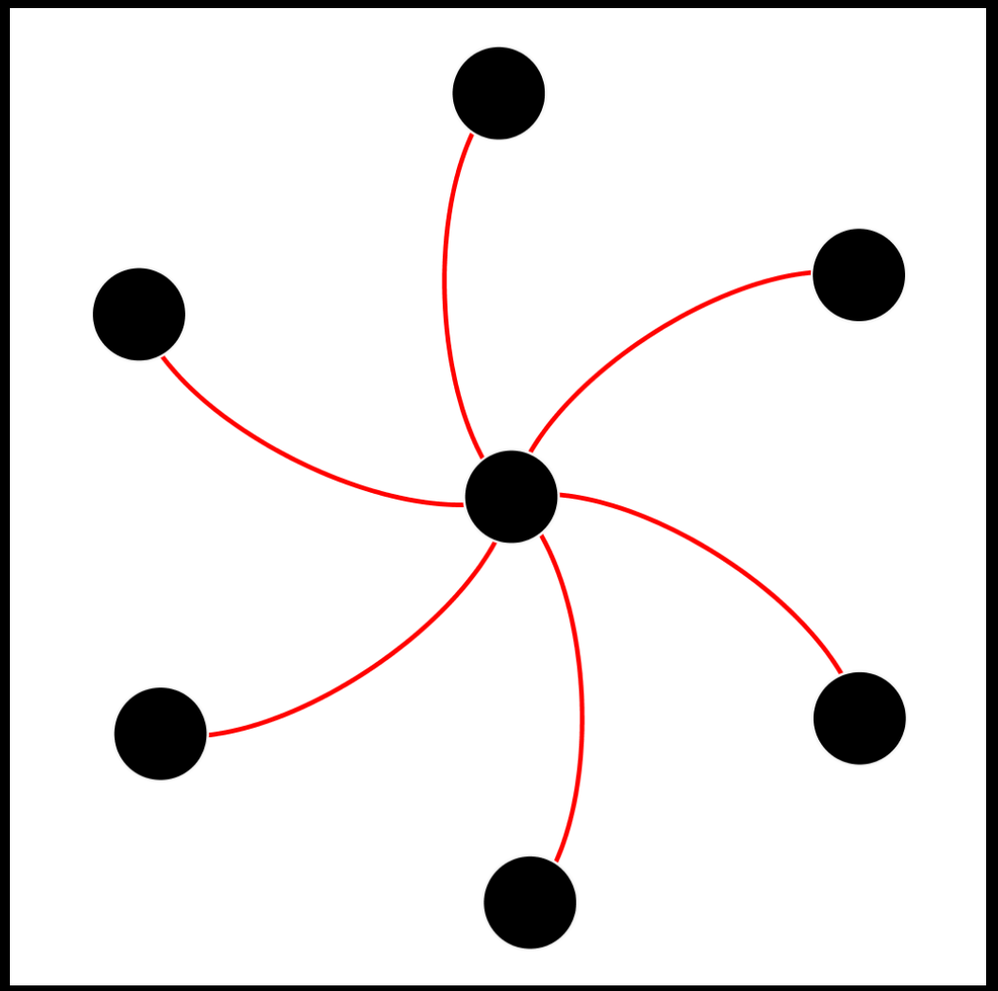
- Undirected graphs
  - degree = number of *incident* edges
- Directed graphs
  - in-degree
  - out-degree

Hubs and Bridges

Hubs


Vivien Leigh
Bryan Cranston
Wilmer Valderrama
George Reeves
Kevin McCarthy
Arnold Schwarzenegger
John Slattery
Ben Kingsley
John Cleese
Ingrid Bergman
June Lockhart
Judy Garland
Roddy McDowall
Eddie Albert
Paul Newman
Humphrey Bogart
Tom Cruise
Cameron Diaz
George Clooney
Julia Roberts
Laura Linney
Clint Eastwood
Sidney Poitier
Ed Asner
Walter Matthau
Kirk Douglas
Lana Turner
Laurie Metcalf
Jack Lemmon
Kevin Spacey
Halle Berry
Gary Sinise
Sally Field
Diane Lane
Burt Lancaster
Emile Hirsch
Kristen Stewart
Forest Whitaker
Denzel Washington
Source: oracleofbacon.org

Hubs



Communities

Bridges

Cynthia Leblanc

Bridges

Bridges

Matthew Hoglie

Tom Holowach

Link Prediction

Measuring Importance

“Centrality”

PANDEMIC

A NEW CHALLENGE

INFECTION DECK

INFECTION DISCARD PILE

INFECTION RATE

2 2 2 3 3 4 4

ST. PETERSBURG
ESSEN
LONDON
MILAN
MOSCOW
PARIS
TEHRAN
ISTANBUL
MADRID
BAGHDAD
DELHI
ALGIERS
KARACHI
KOLKATA
CAIRO
RIYADH
MUMBAI
LAGOS
KHARTOUM
CHENNAI
KINSHASA
JOHANNESBURG

SEOUL
BEIJING
TOKYO
SHANGHAI
HONG KONG
OSAKA
TAIPEI
BANGKOK
MANILA
HO CHI MINH CITY
JAKARTA
SYDNEY

CHICAGO
MONTRÉAL
NEW YORK
SAN FRANCISCO
WASHINGTON
ATLANTA
LOS ANGELES
MIAMI
MEXICO CITY
BOGOTA
LIMA
SÃO PAULO
BUENOS AIRES
SANTIAGO

OUTBREAKS

0 1 2 3 4 5 6 7

CURED DISEASES/FLIP TOKEN IF ERADICATED

PLAYER DECK

PLAYER DISCARD PILE

PLAY

1. Do 4 actions
2. Draw 2 cards
   - Resolve any epidemics
   - Discard to 7 cards
3. Infect cities

Defining “Centrality”

Defining “Centrality”

"There is certainly no unanimity on exactly what centrality is or on its conceptual foundations, and there is little agreement on the proper procedure for its measurement.”
-Freeman, 1979

Defining “Centrality”

- Influence
- Prestige
- Power
- Control

Degree Centrality

Adj Matrix A =

$$\begin{bmatrix} 1 & 0 & 1 & 0 & \dots & a_{1j} & \dots & a_{1n} \\ \vdots & & & \vdots & & \vdots & & \\ & & & & & & & \\ a_{n1} & \dots & a_{nj} & \dots & \dots & \dots & 1 \end{bmatrix}$$

$$\mathbf{c}_i^d = \sum_{i=1}^{n} a_i$$

$$\bar{c}_i^d = \frac{c_i^d}{n-1}$$

Closeness Centrality

Farness =>

$$\text{farness}(v_i) = \sum_{j \neq i} \text{shortest} \ldots \text{ance}(v_i, v_j)$$

Only defined for connected components

Closeness =>

$$\frac{1}{\text{farness}(v_i)}$$

Closeness =>

$$\frac{n-1}{\text{farness}(v_i)}$$

Maximum closeness

Betweenness Centrality

$$Betwenness(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

Undirected

$$\overline{Betweenness}(v) = \frac{2 \times Betweenness(v)}{(n-1)(n-2)}$$

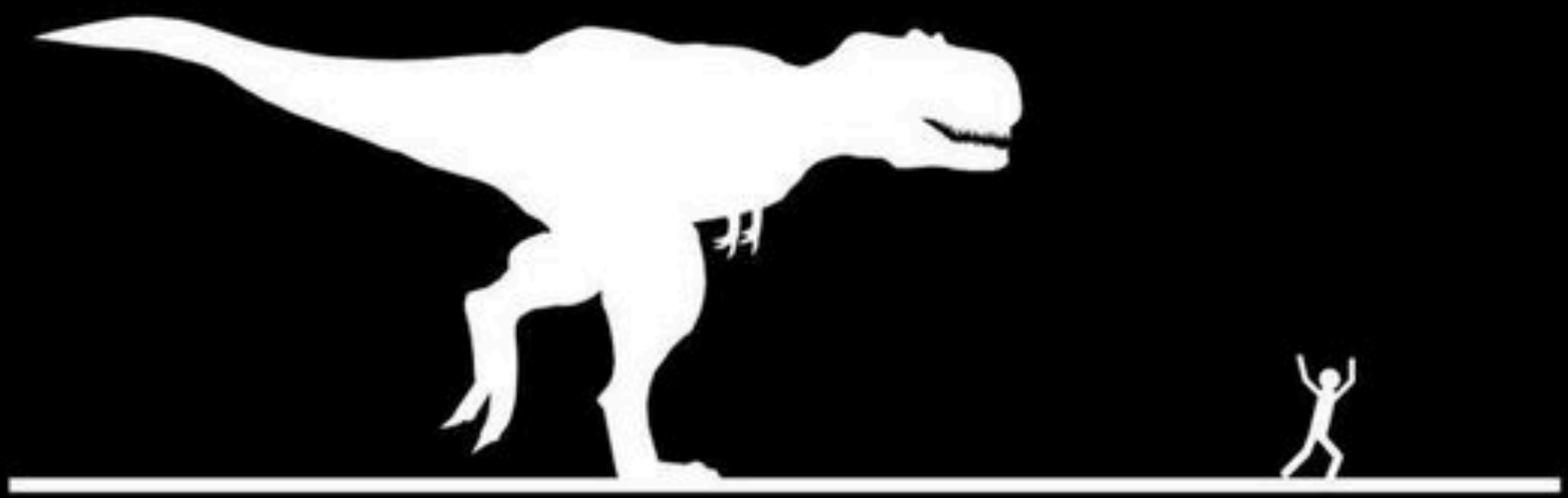
Directed

$$\overline{Betweenness}(v) = \frac{Betweenness(v)}{(n-1)(n-2)}$$

Eigenvector Centrality

$$A\pi = \lambda\pi$$

π is the eigenvector
π_i = eigenvector centrality for a_i

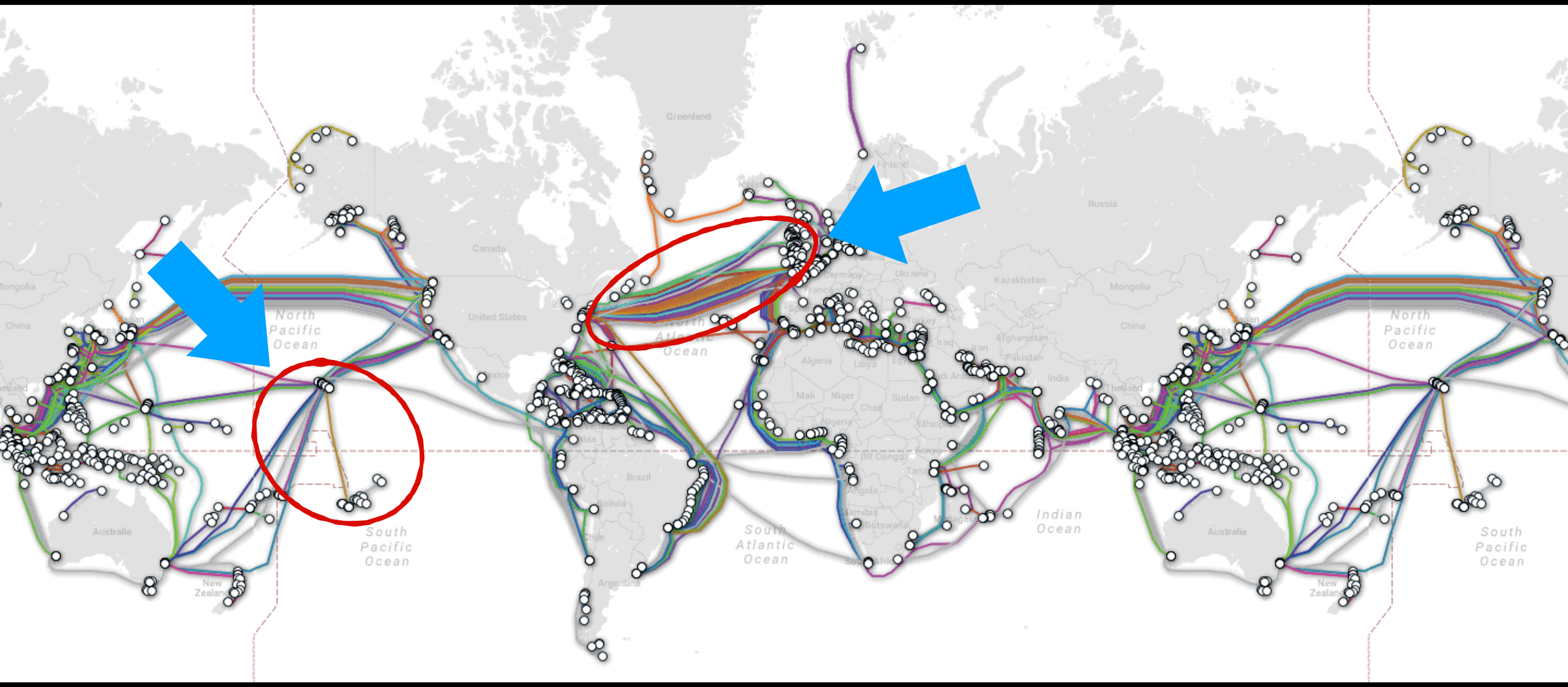


EXERCISE

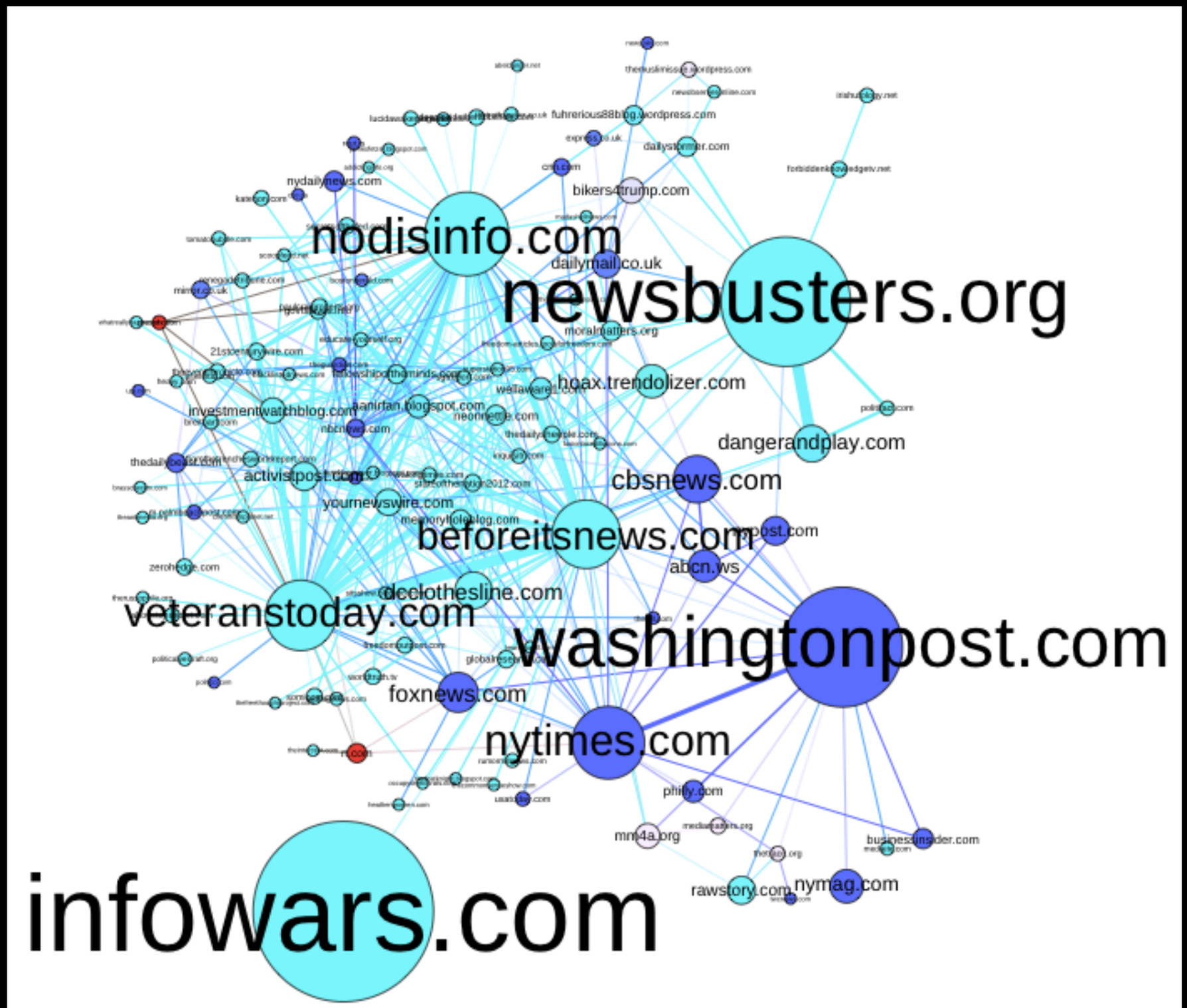
Some motivation required.

We Will Now Have A
10 MINUTE
INTERMISSION
BEFORE STARTING OUR
NEXT SHOW

How Connected Are We?



How Connected Are We?

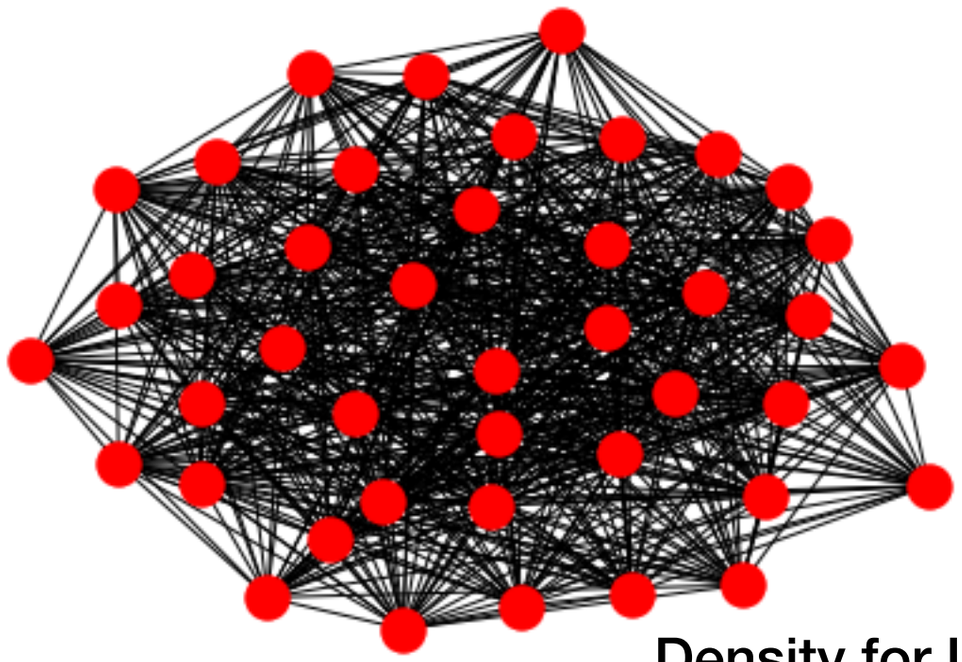


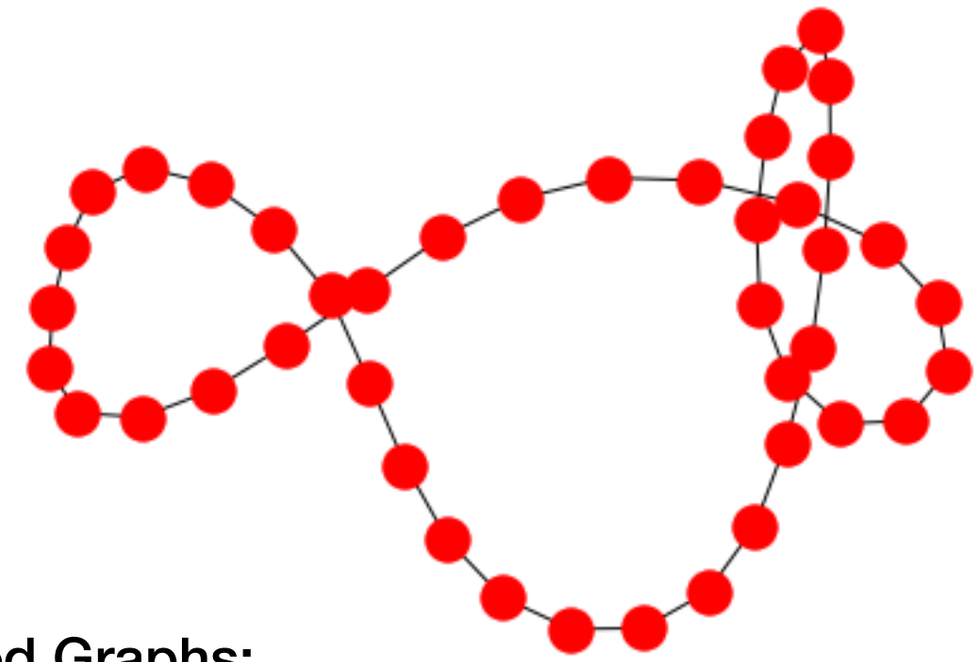
How Connected Are We?

How Connected Are We?

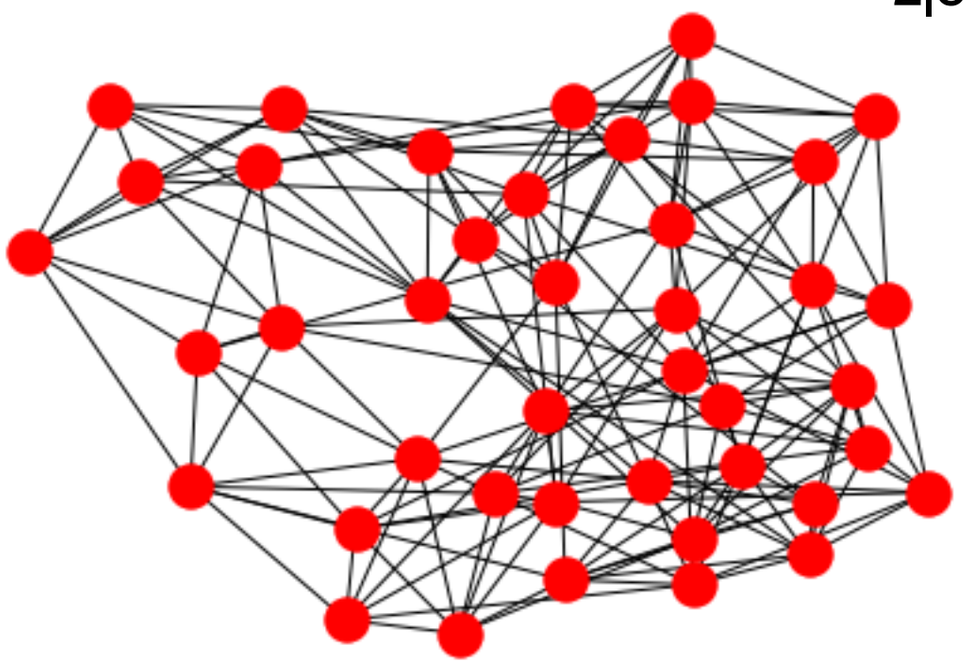
Review

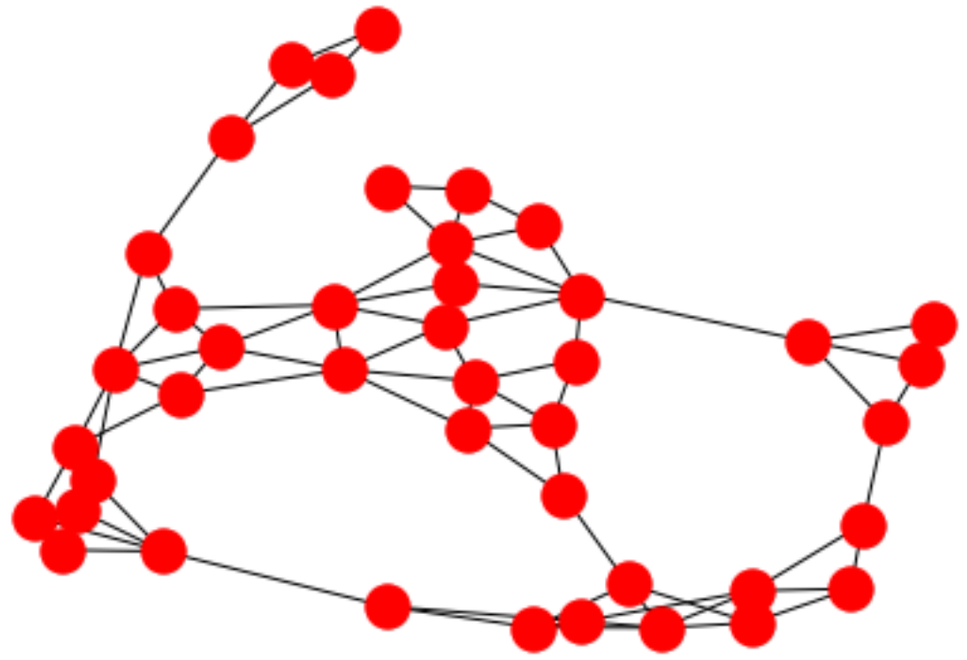
Network Density



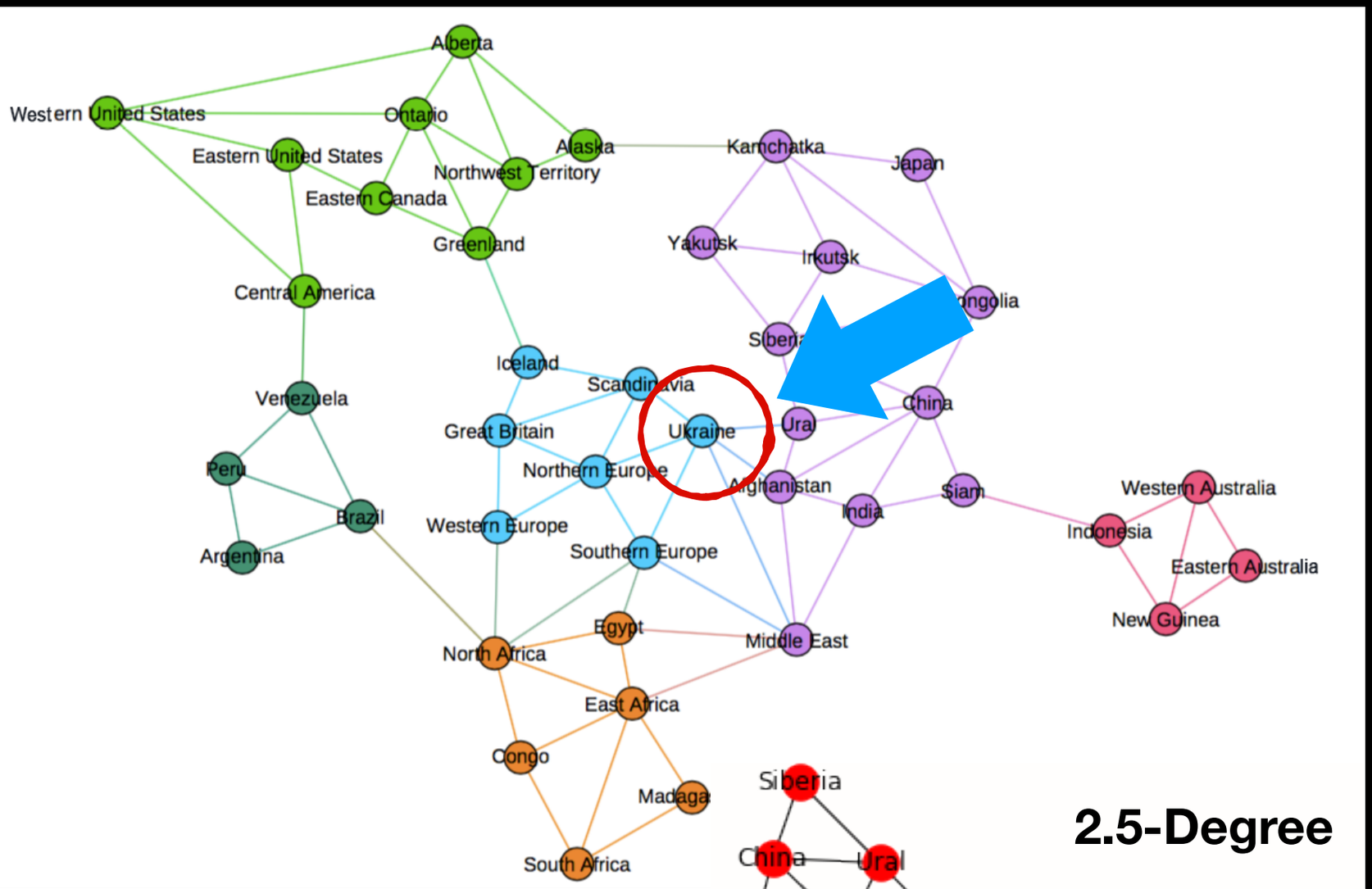


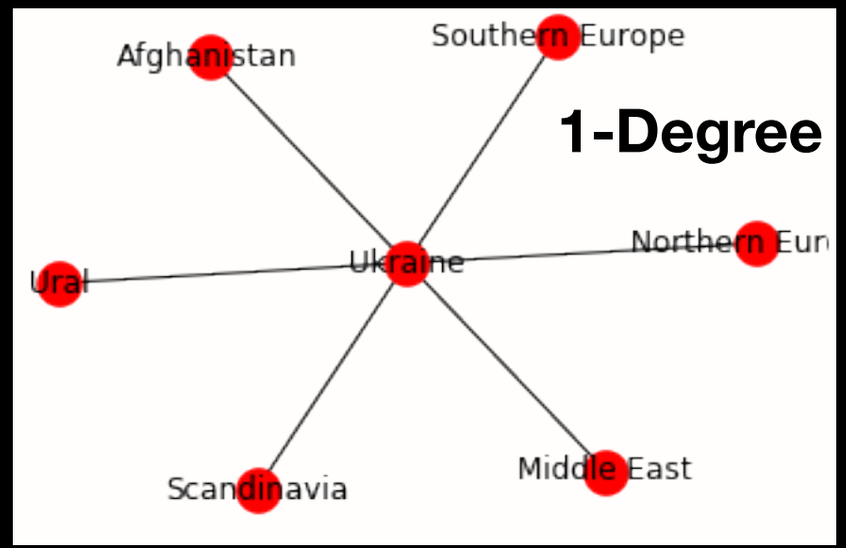
Density for Undirected Graphs:
2|e| / n(n-1)

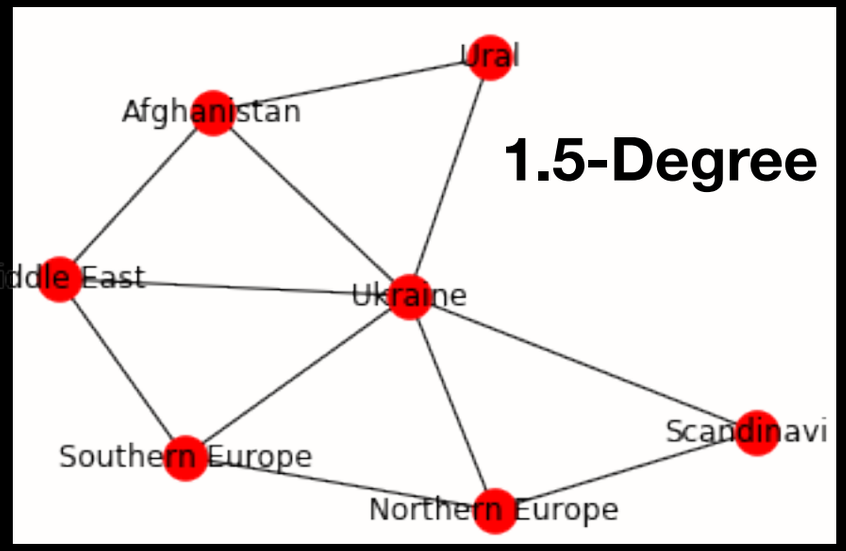


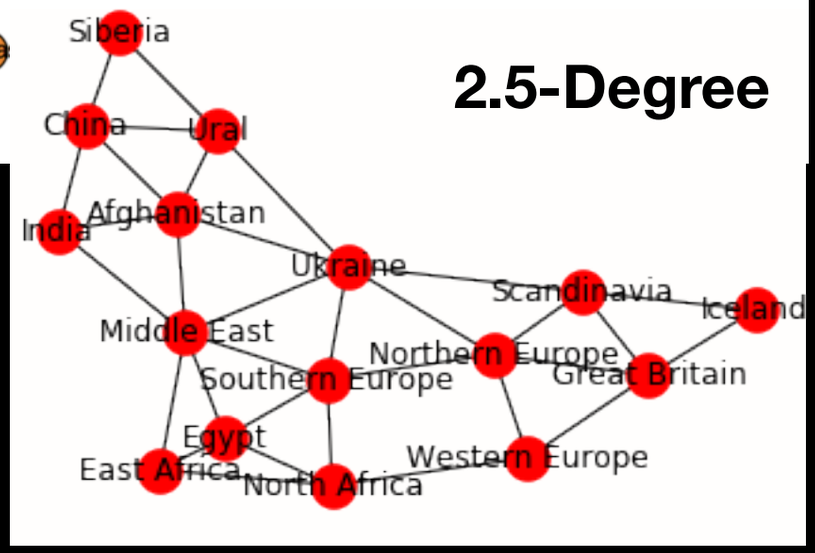


Egocentric Networks

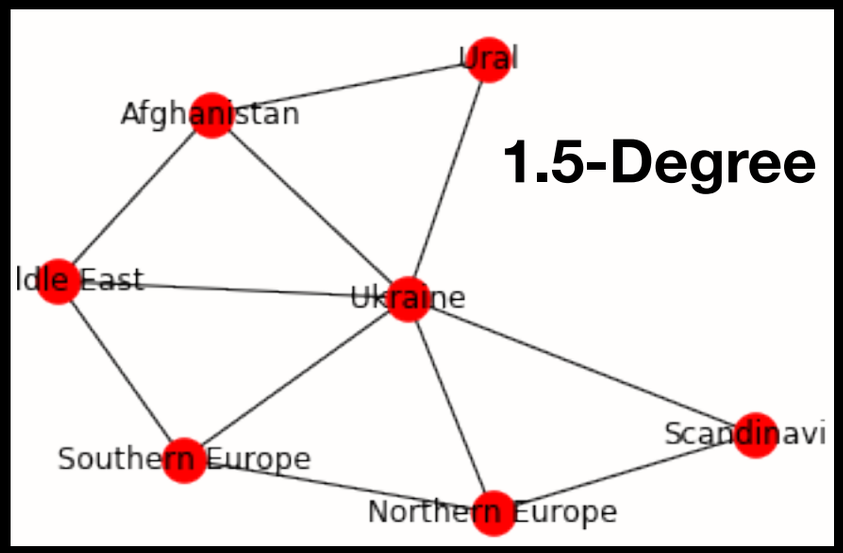


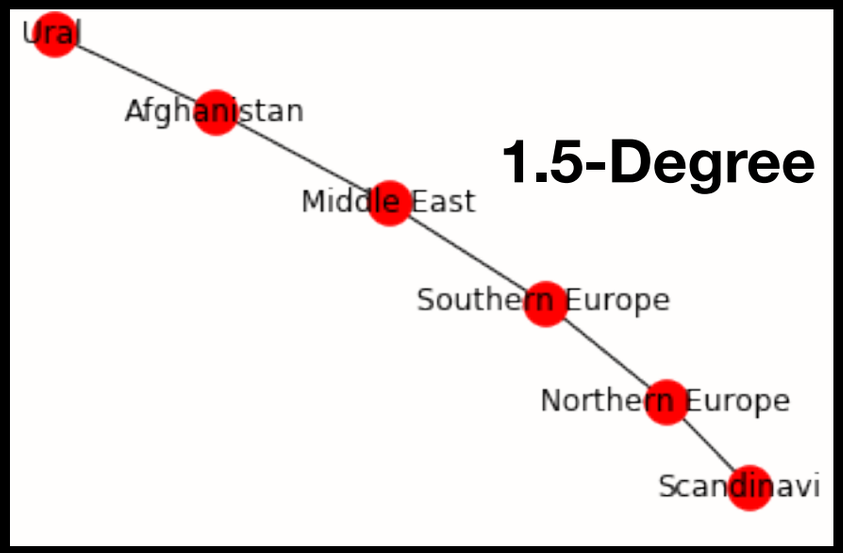






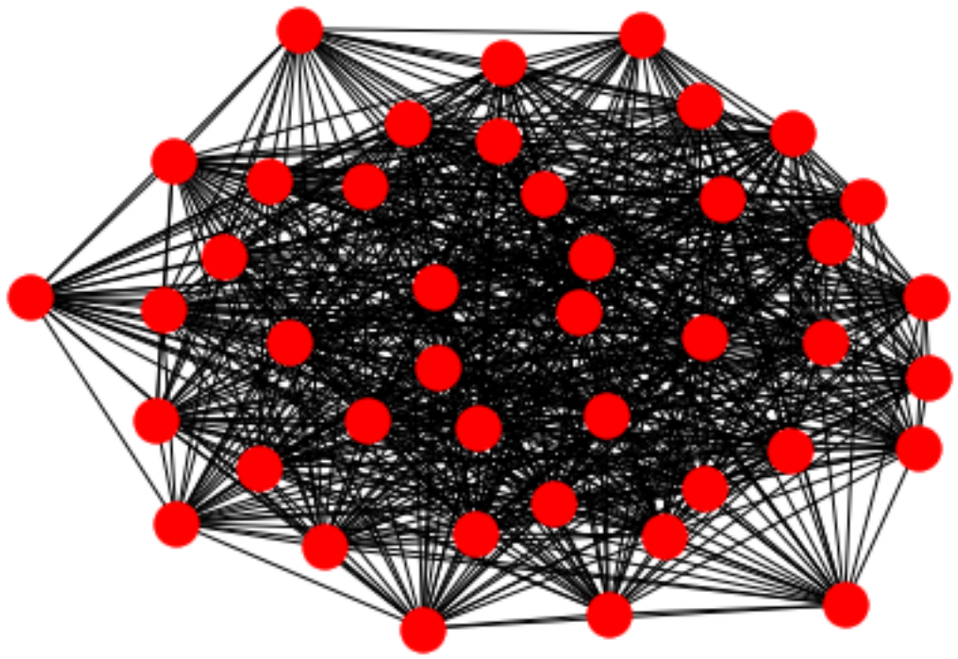
Egocentric Networks

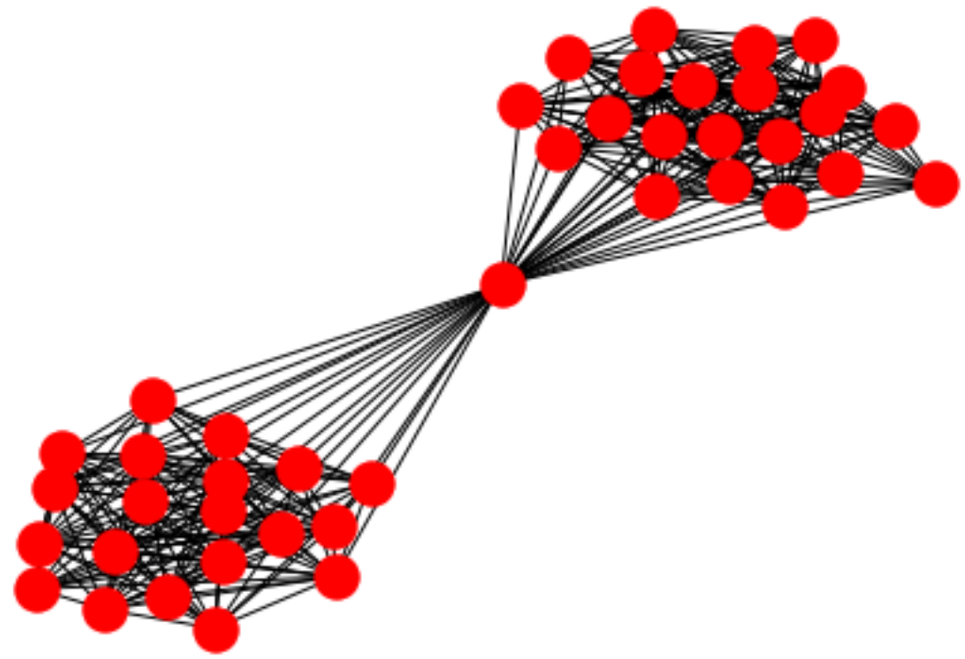




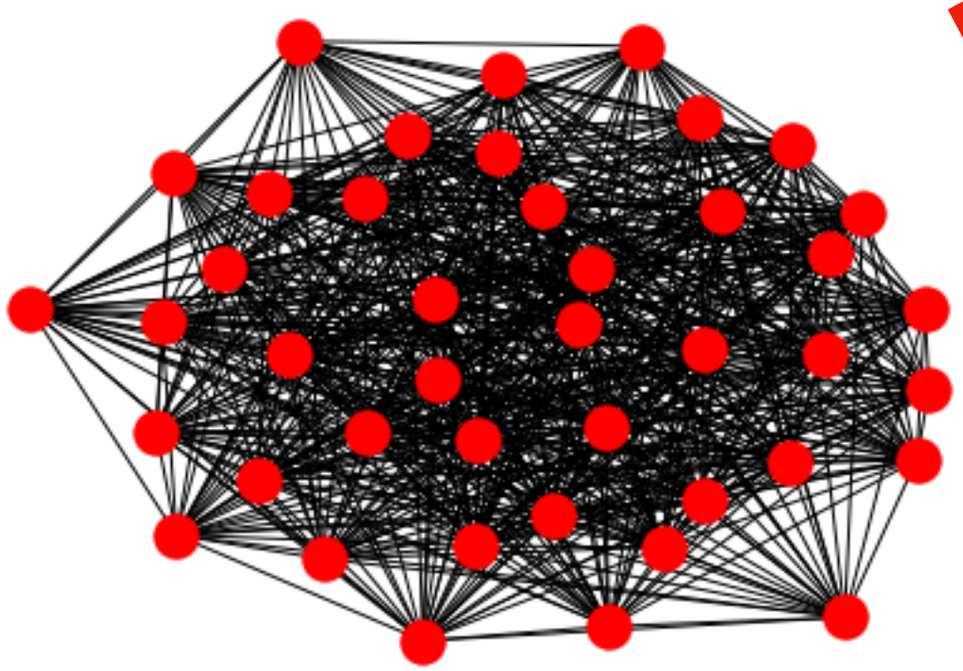
Properties of Graphs

How **Connected** are These Graphs?

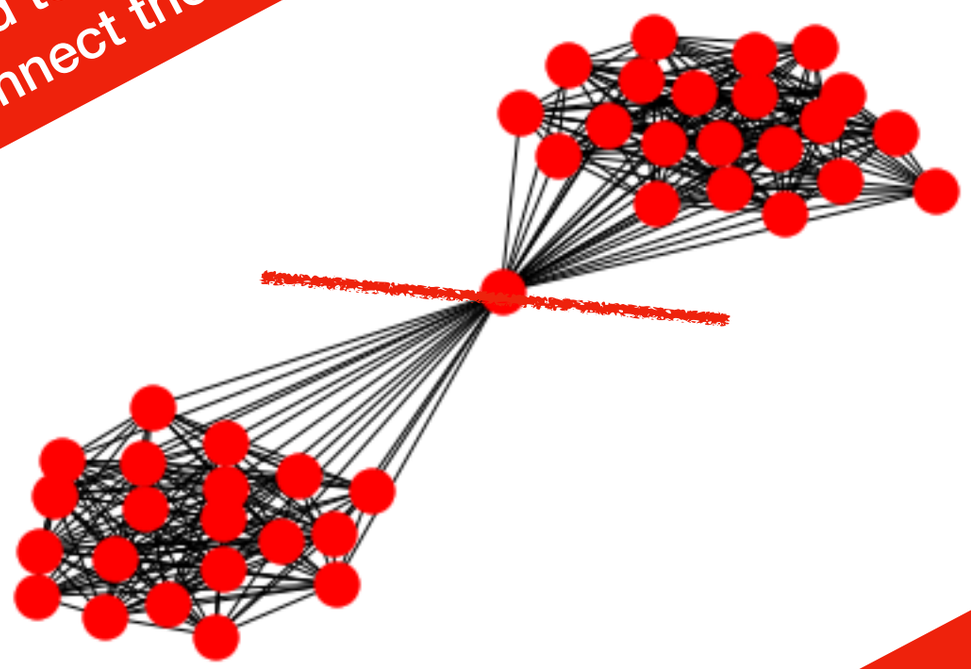




Connectivity

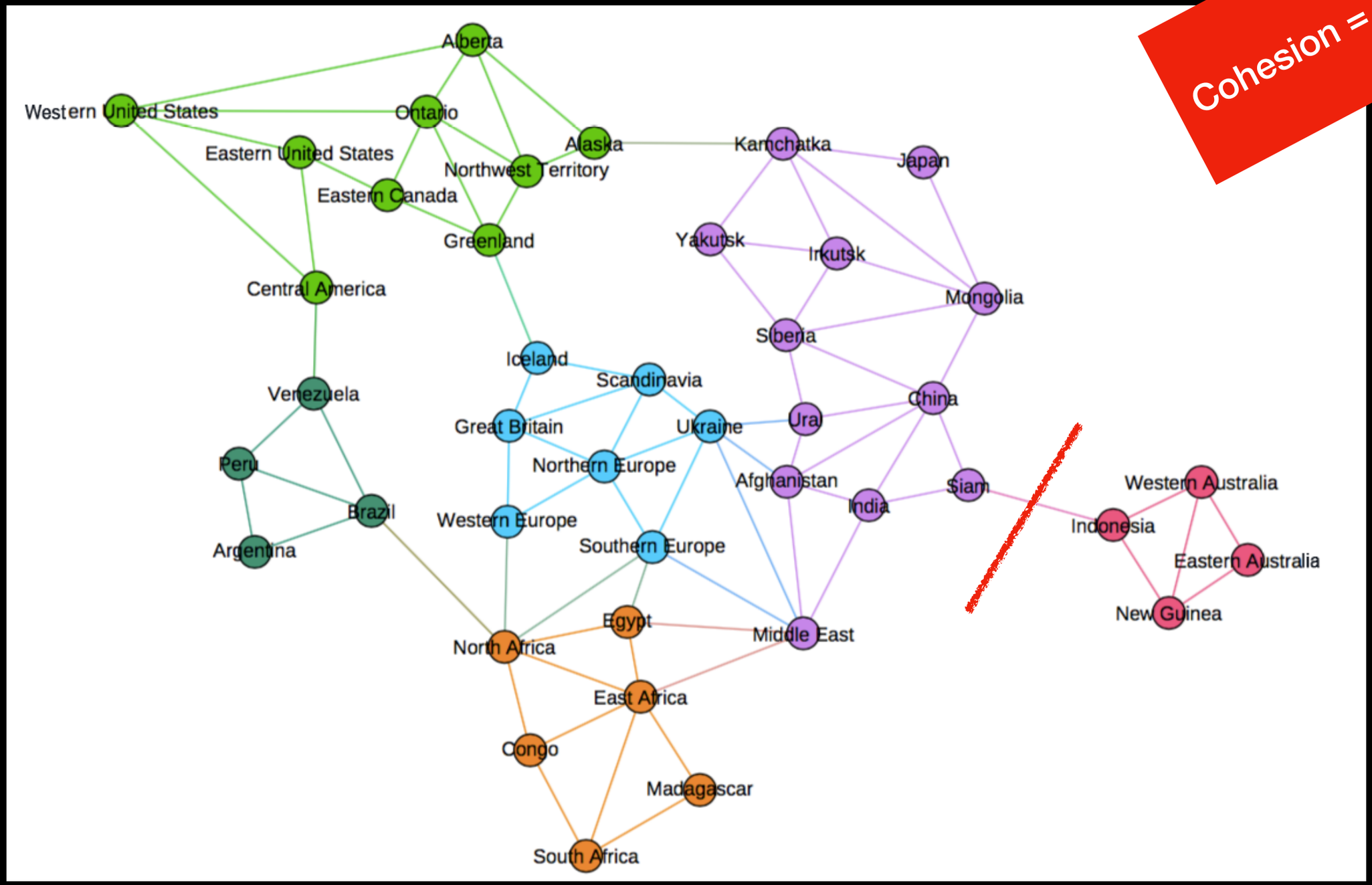


How many nodes do we need to delete to disconnect the graph?

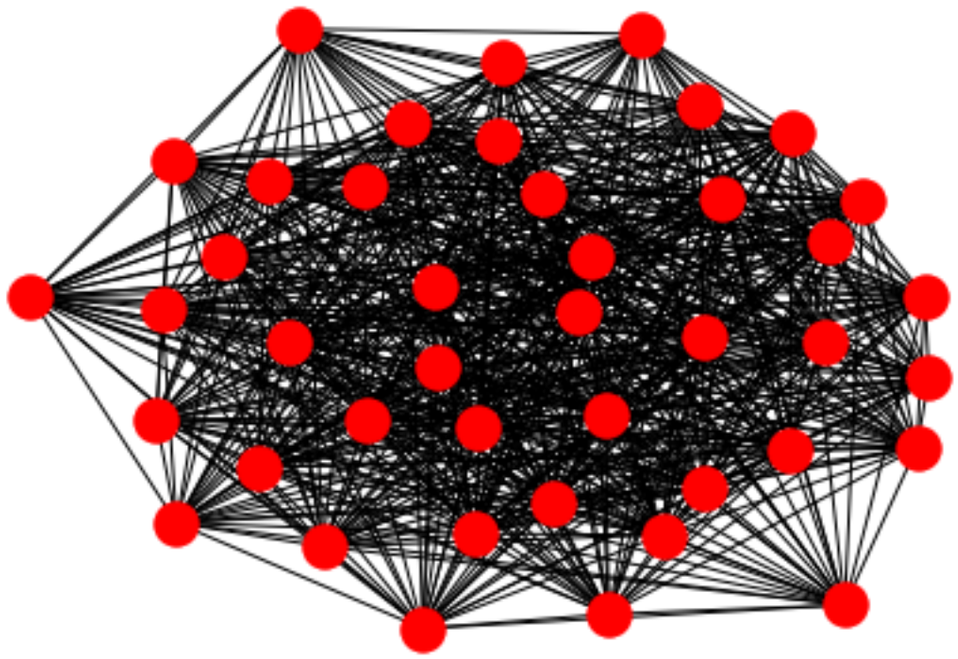


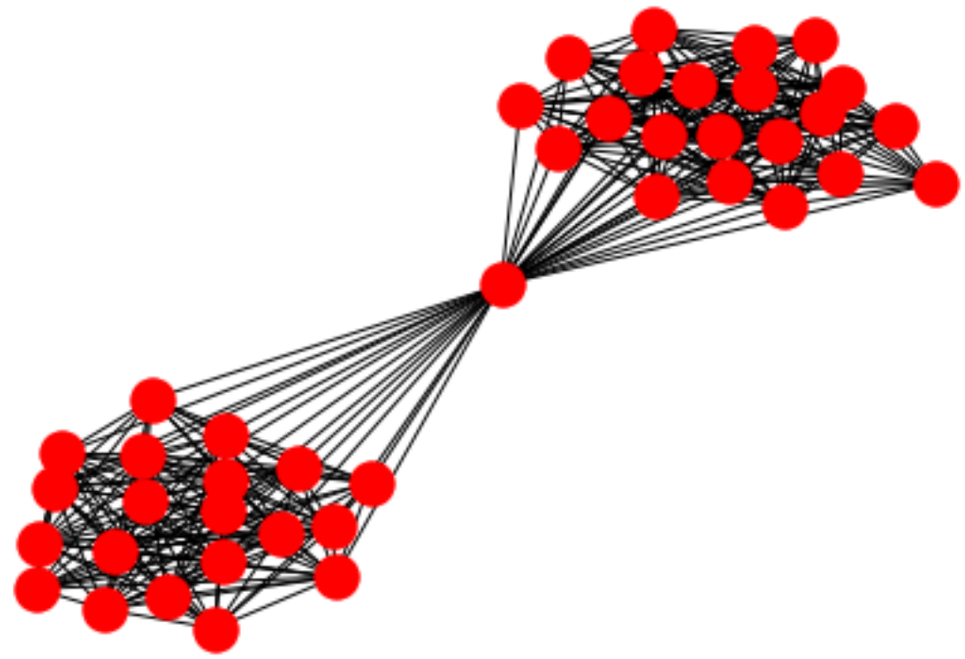
Also called "cohesion"

Connectivity



How Tightly Clustered are These Graphs?





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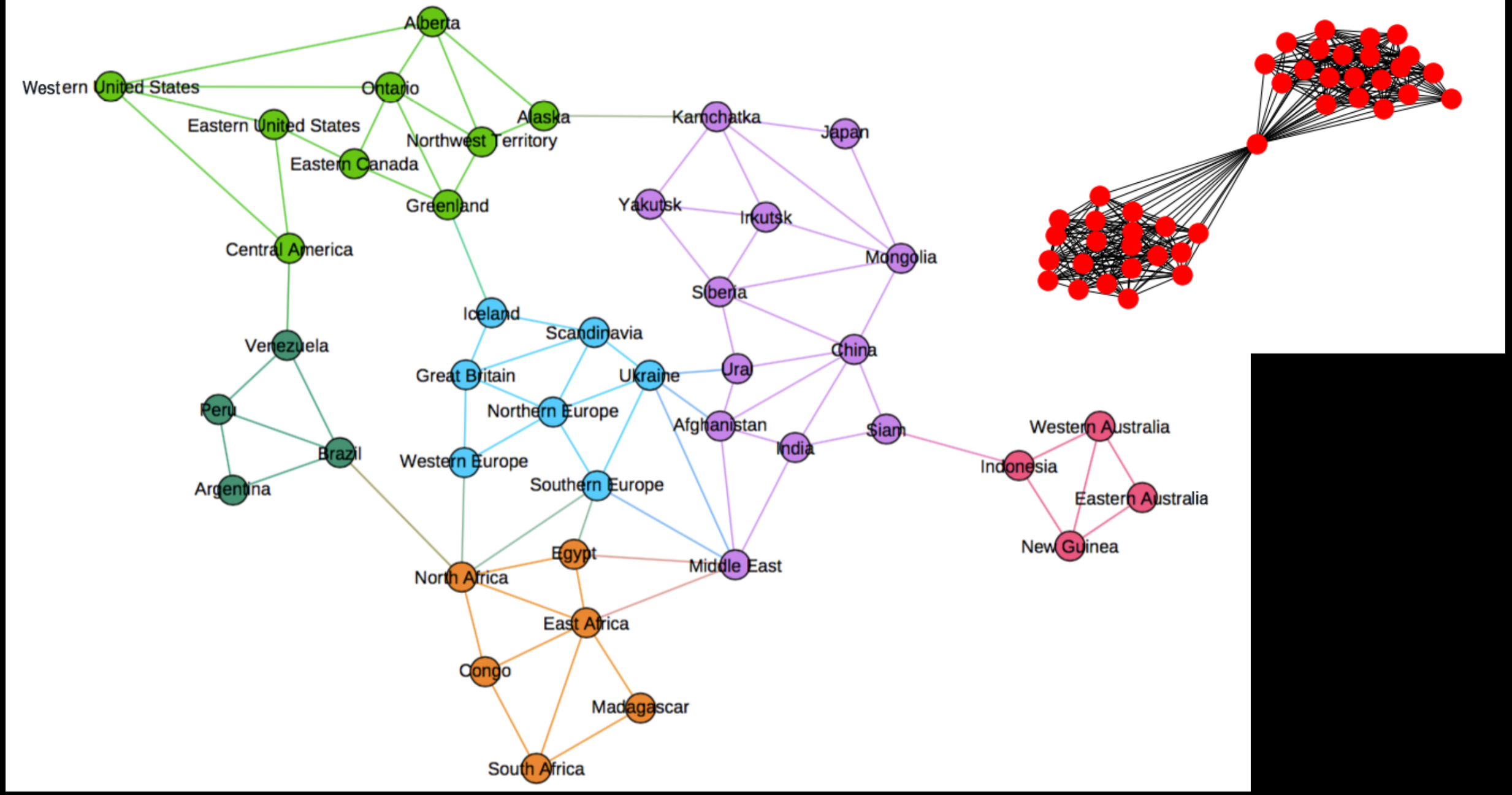
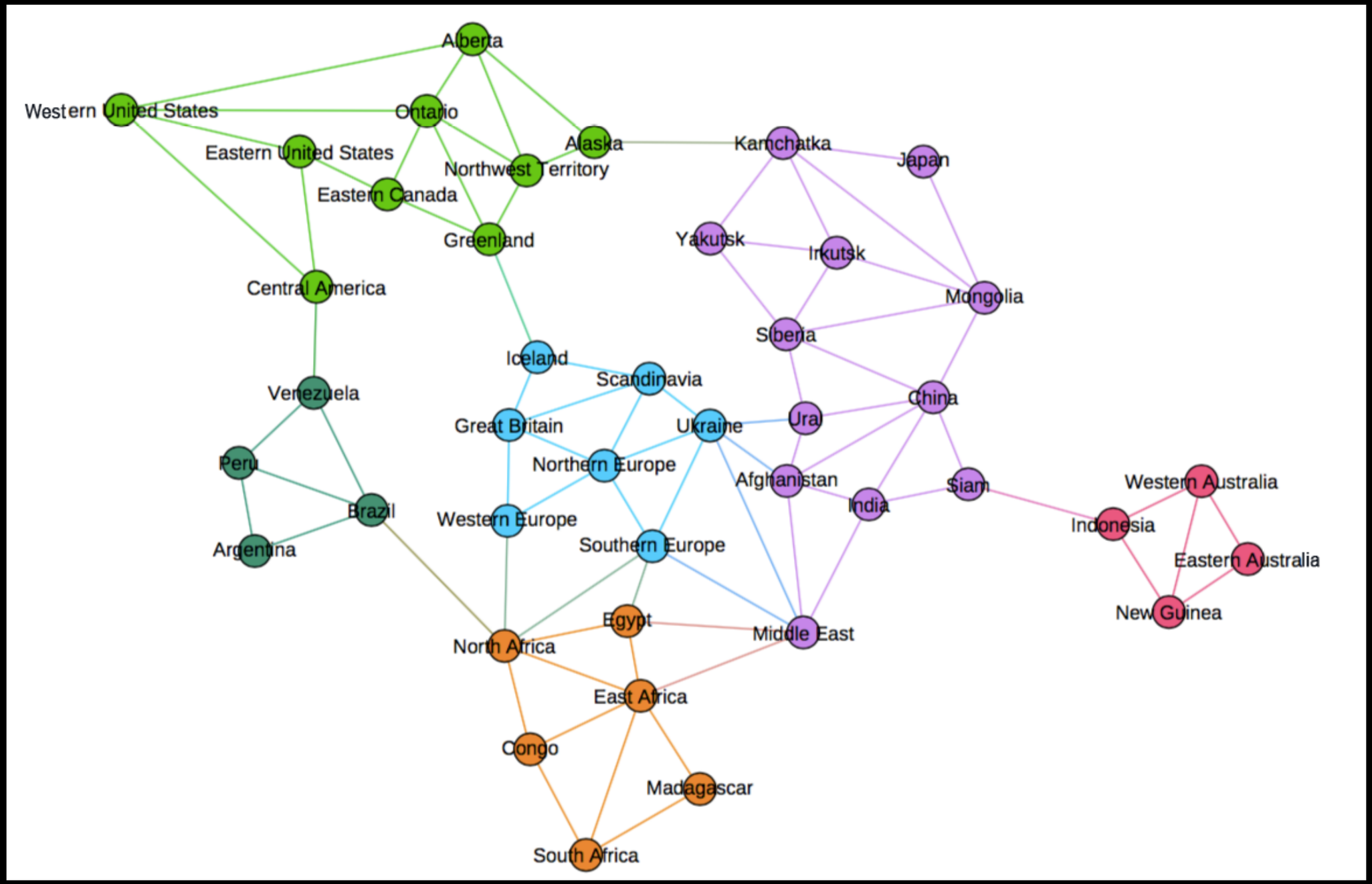
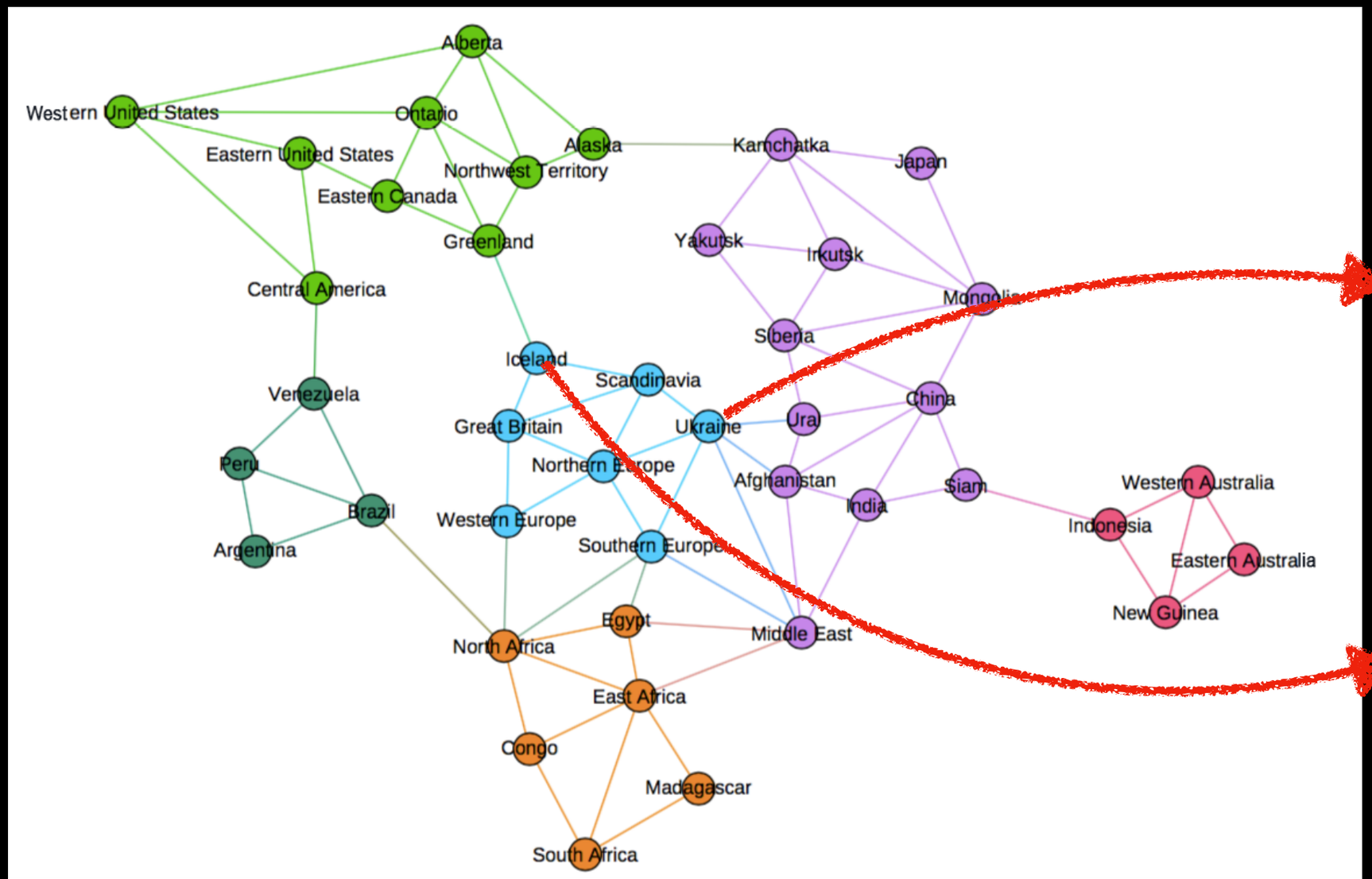
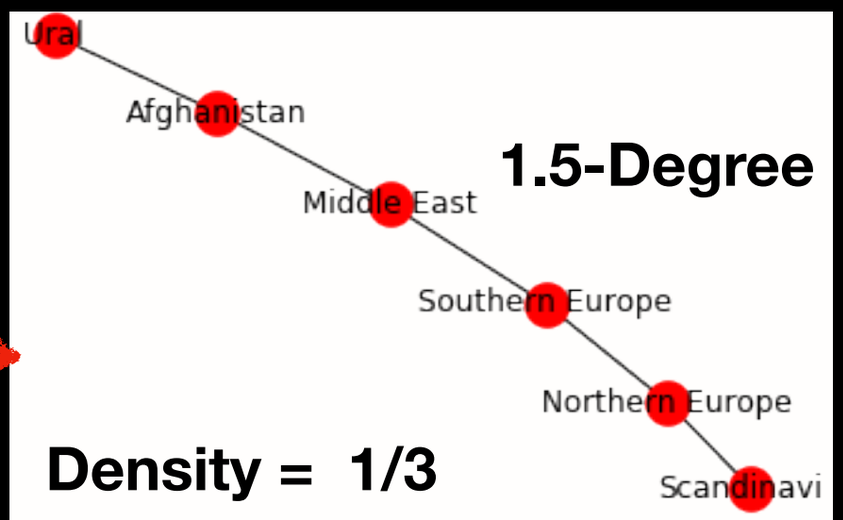


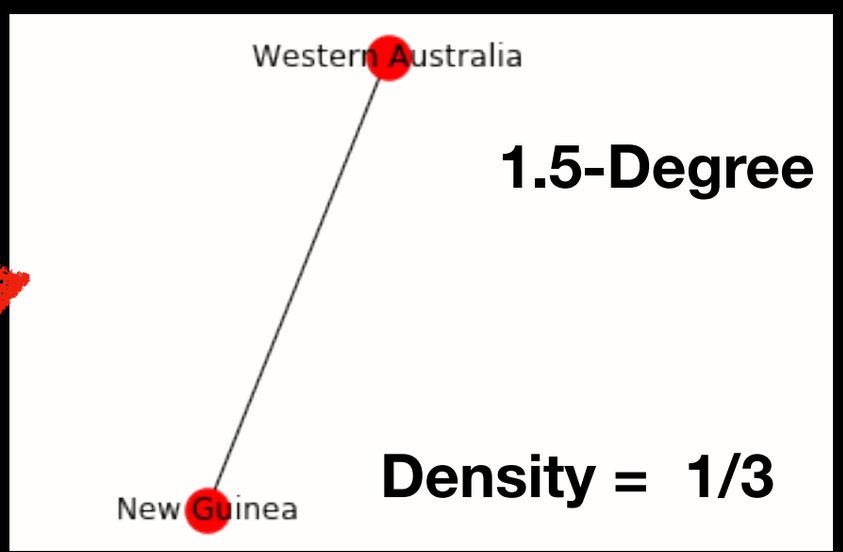
How Tightly Clustered is this Graph?



Density in Egocentric Networks





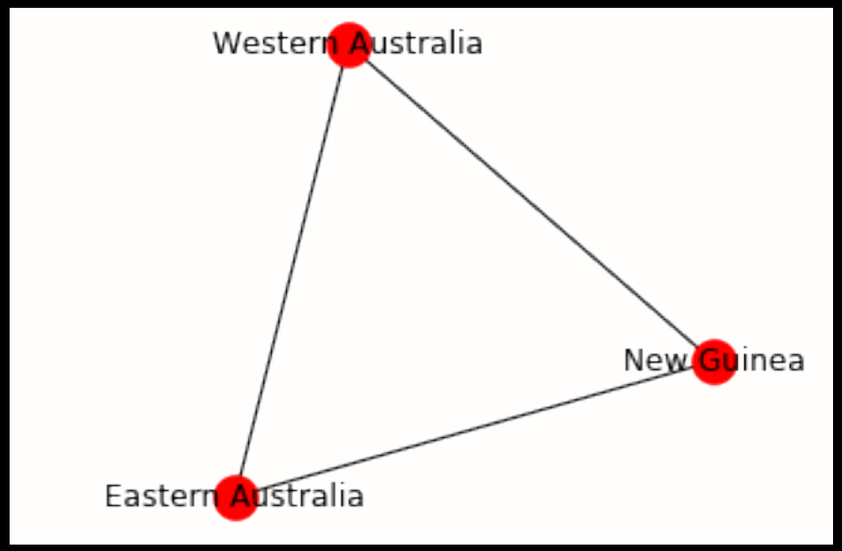


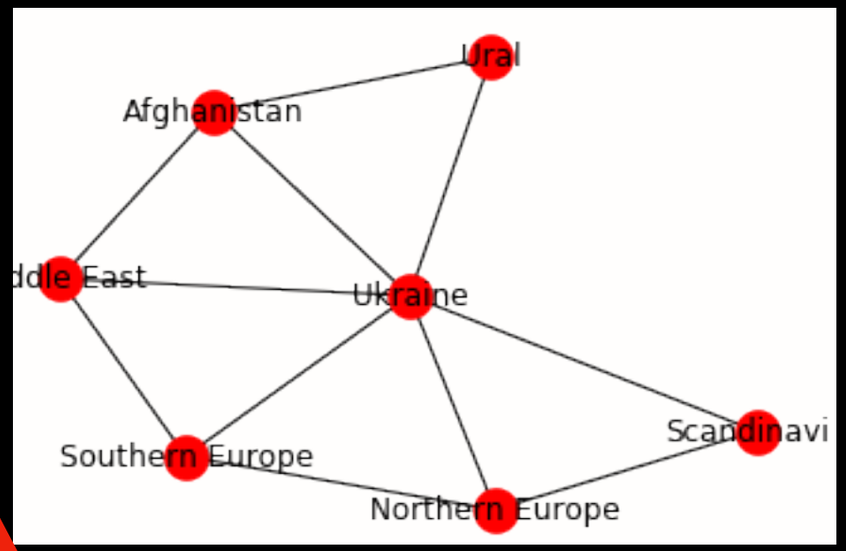
Local Clustering Coefficient

Brazil

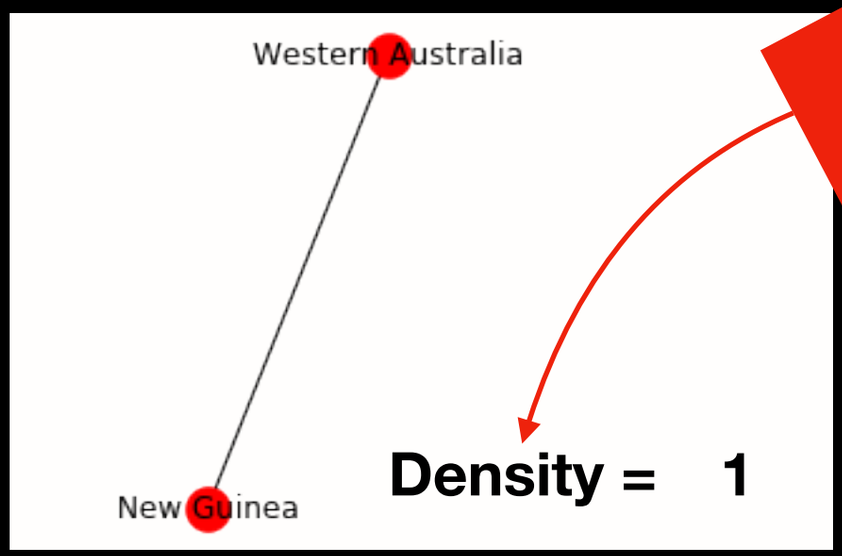
Ukraine

1.5-Degree Ego-Net

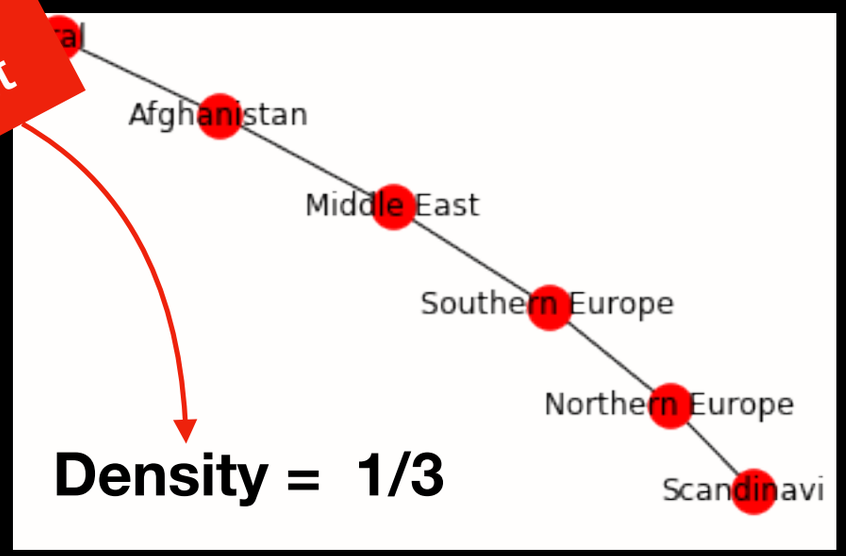




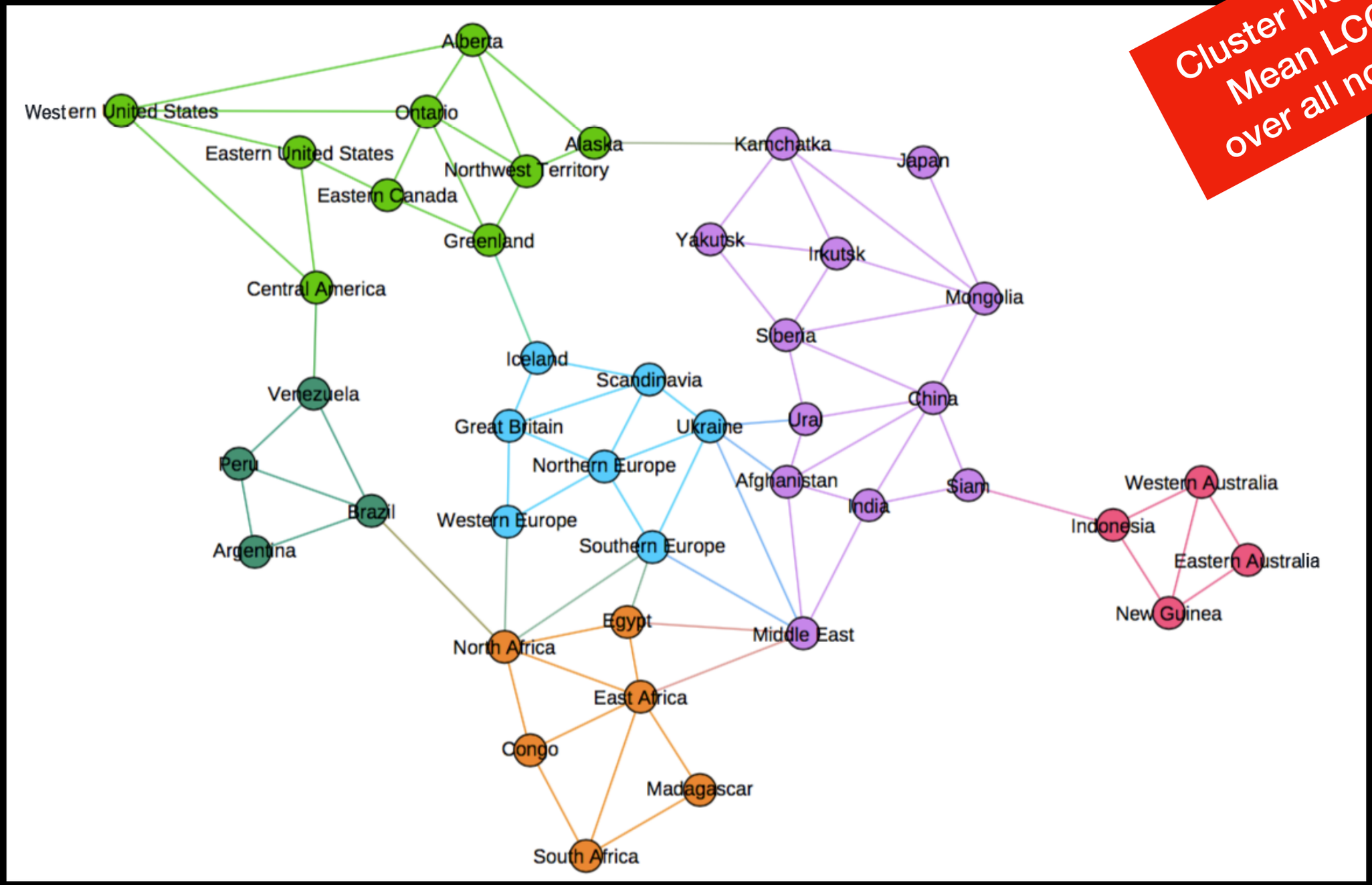
1.5-Degree Ego-Net / target



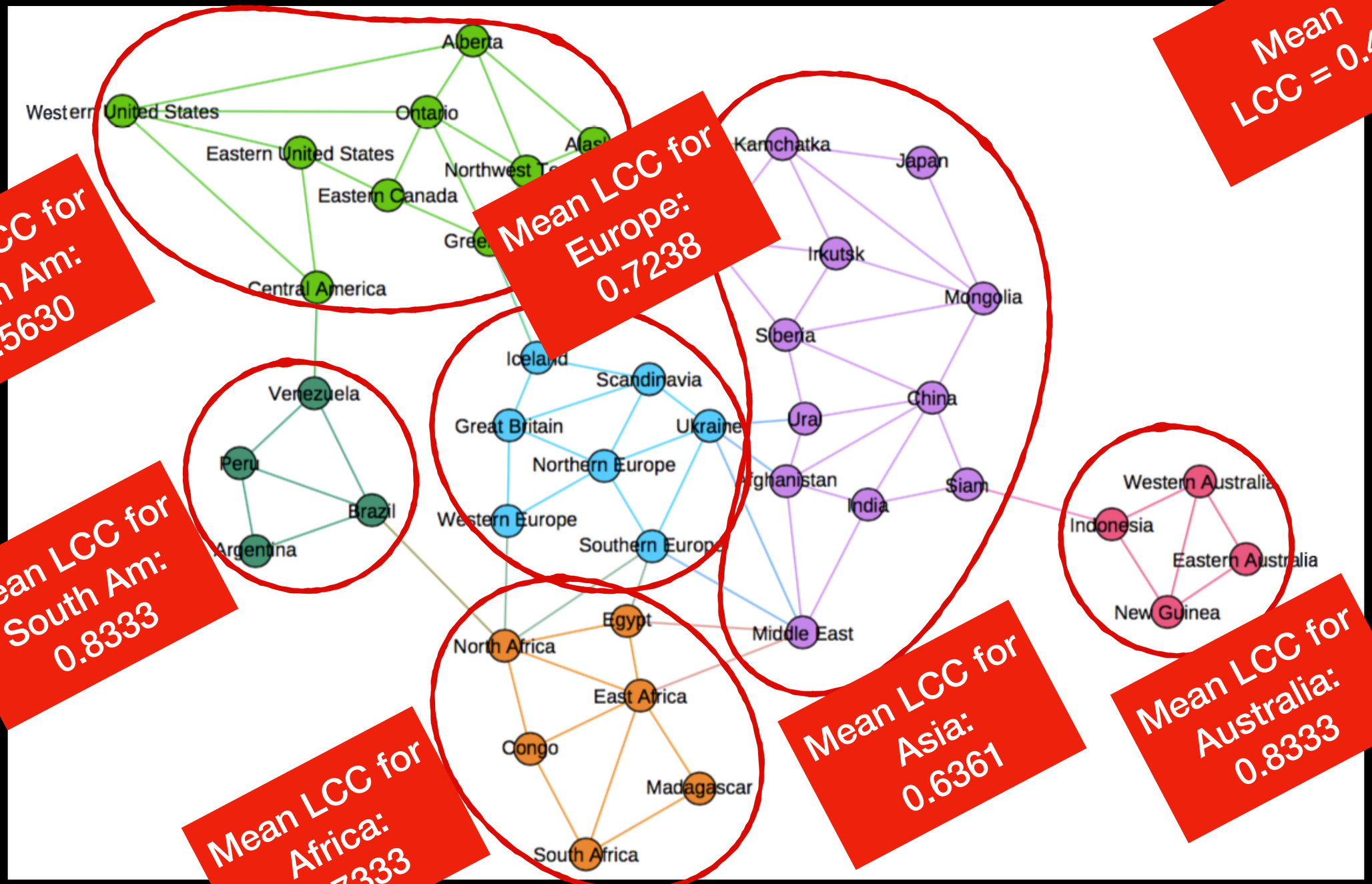
Local Clustering Coefficient



How Tightly Clustered is this Graph?

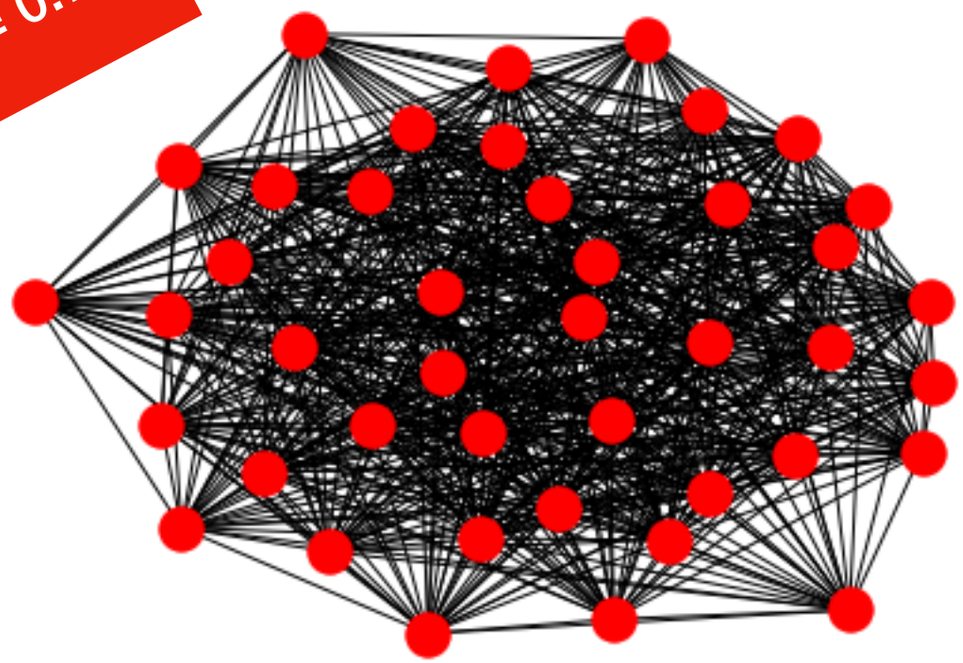


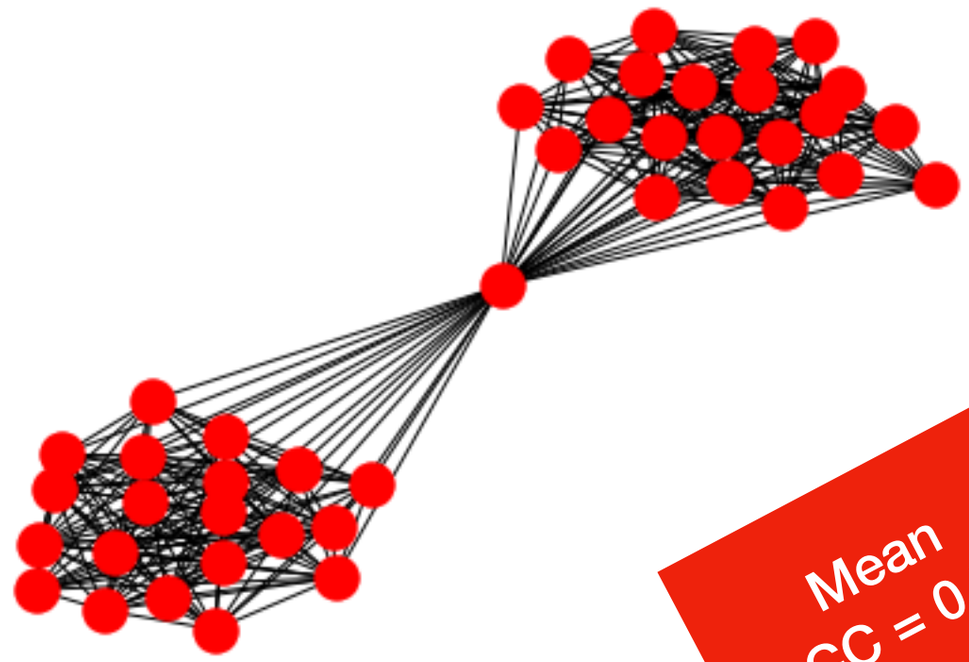
How Tightly Clustered is this Graph?



How Tightly Clustered are These Graphs?

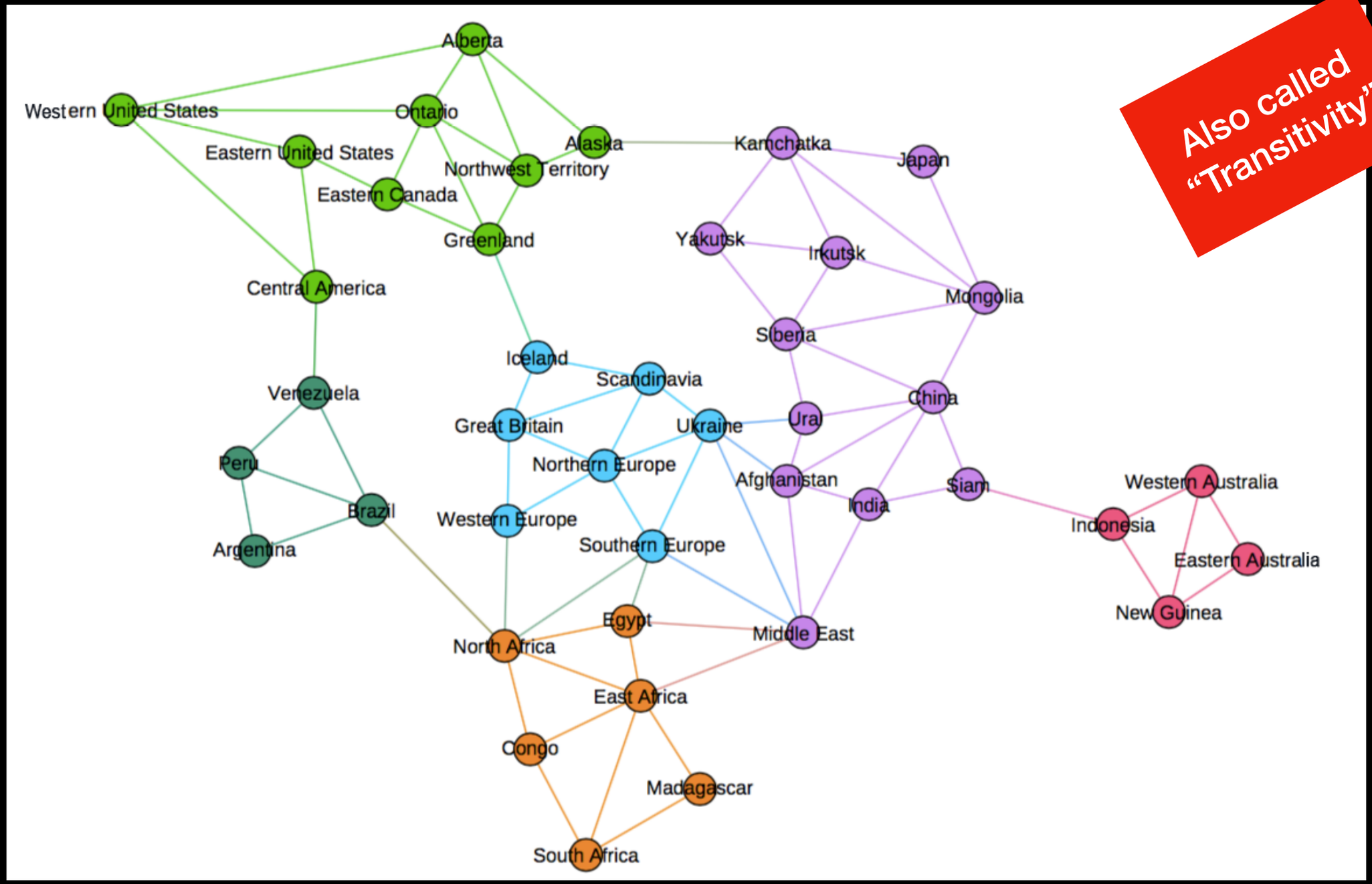
Mean
LCC = 0.74





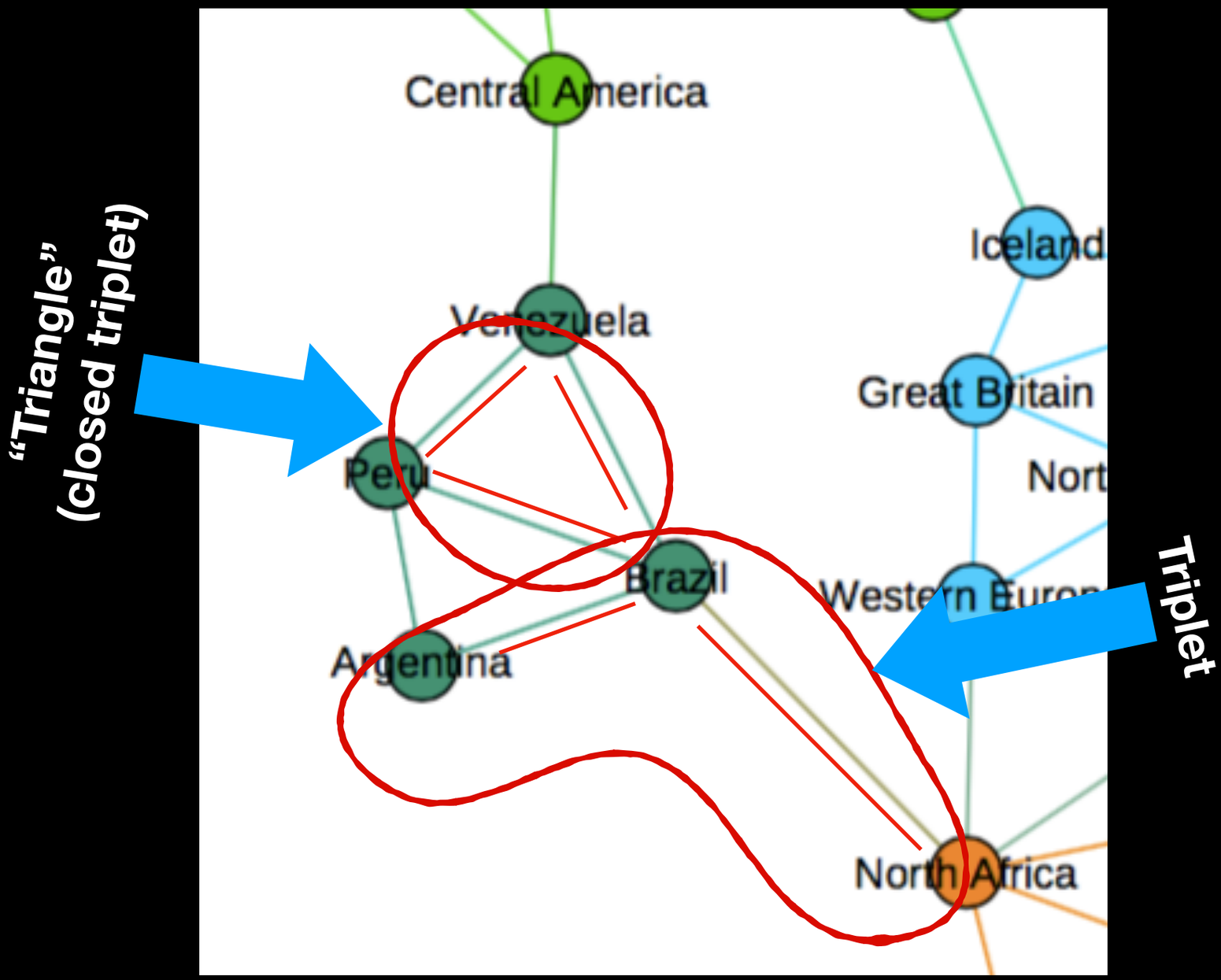
Mean
LCC = 0.71

Another Cluster Metric: Global Clustering Coefficient



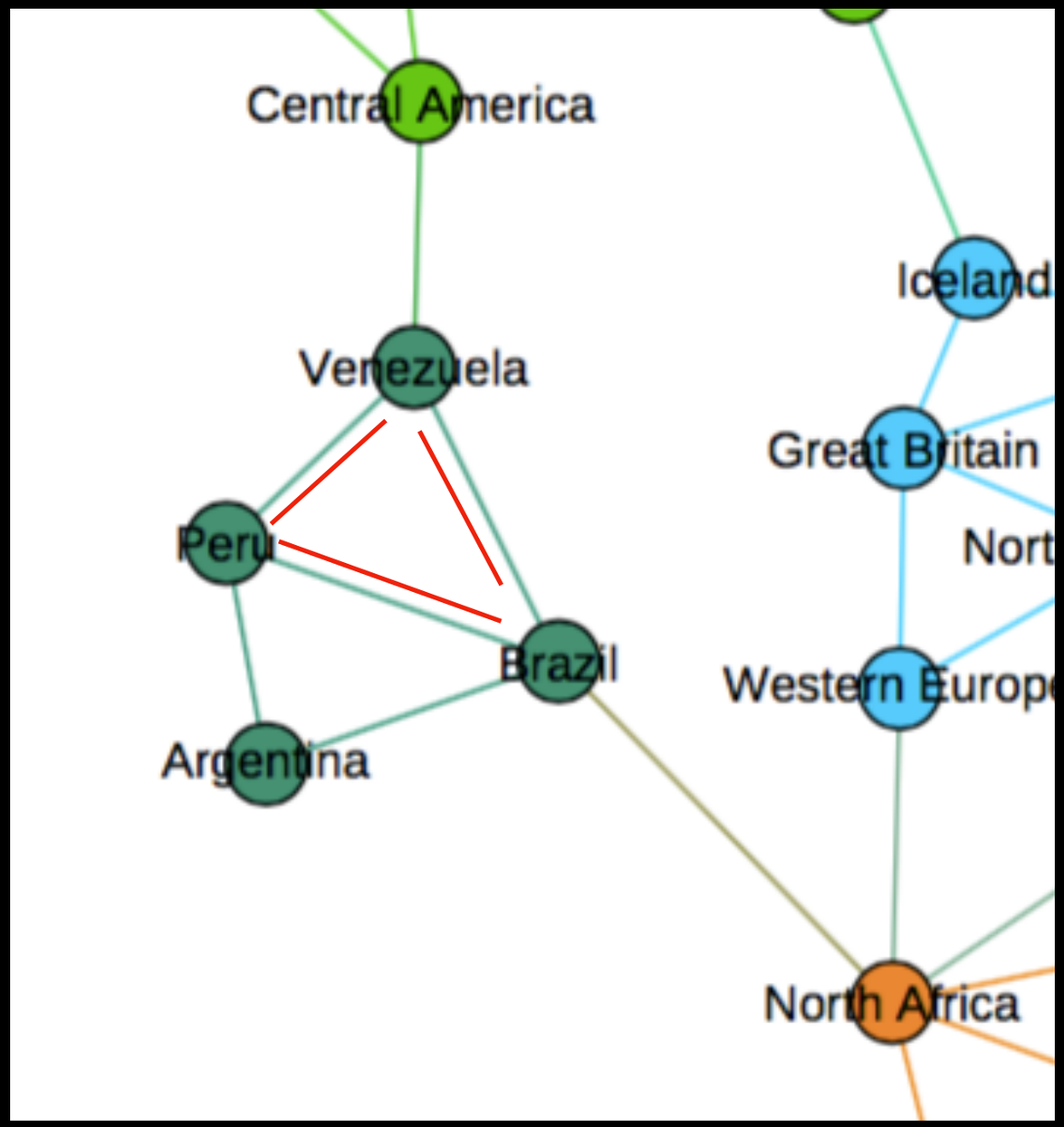
Also called "Transitivity"

Another Cluster Metric: Global Clustering Coefficient



Another Cluster Metric: Global Clustering Coefficient

How many triplets are in a triangle?

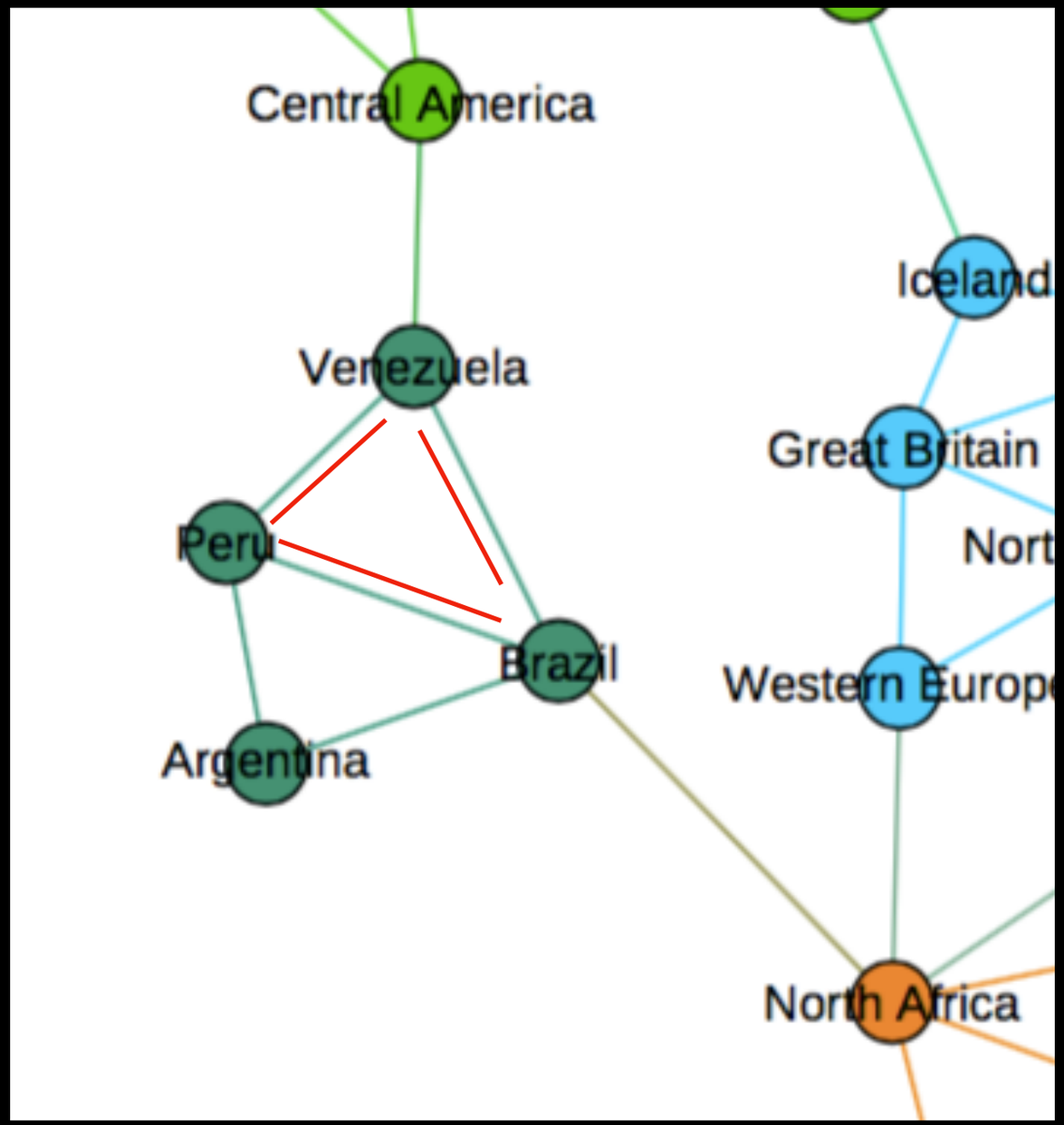


Another Cluster Metric: Global Clustering Coefficient

Global Clustering Coef:

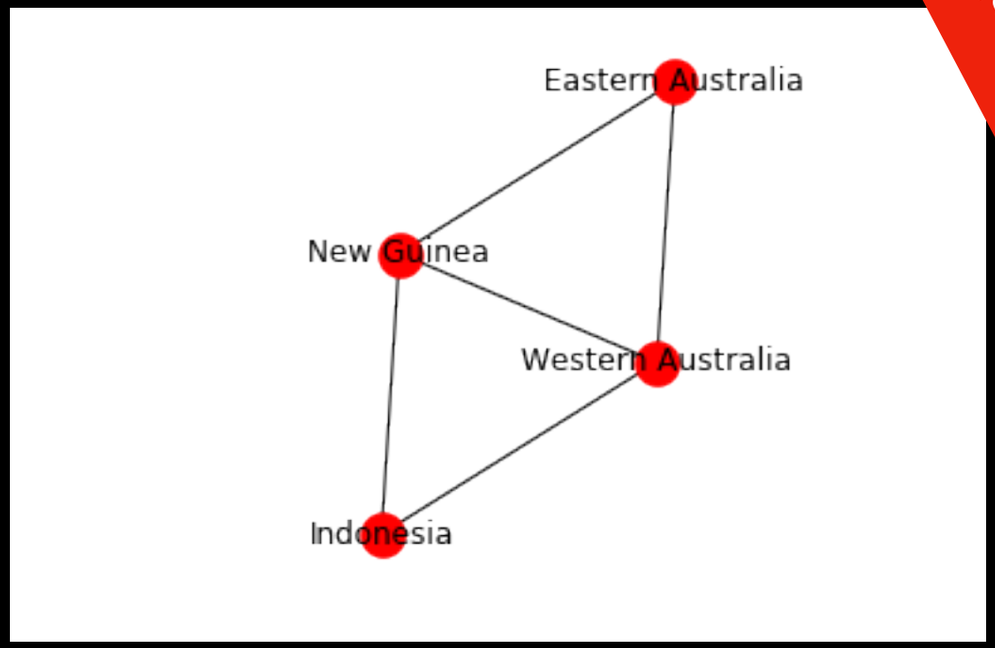
$$\frac{3 \times \text{number of triangles}}{\text{number of triples}}$$

Essentially, how many triplets are closed?



Another Cluster Metric: Global Clustering Coefficient

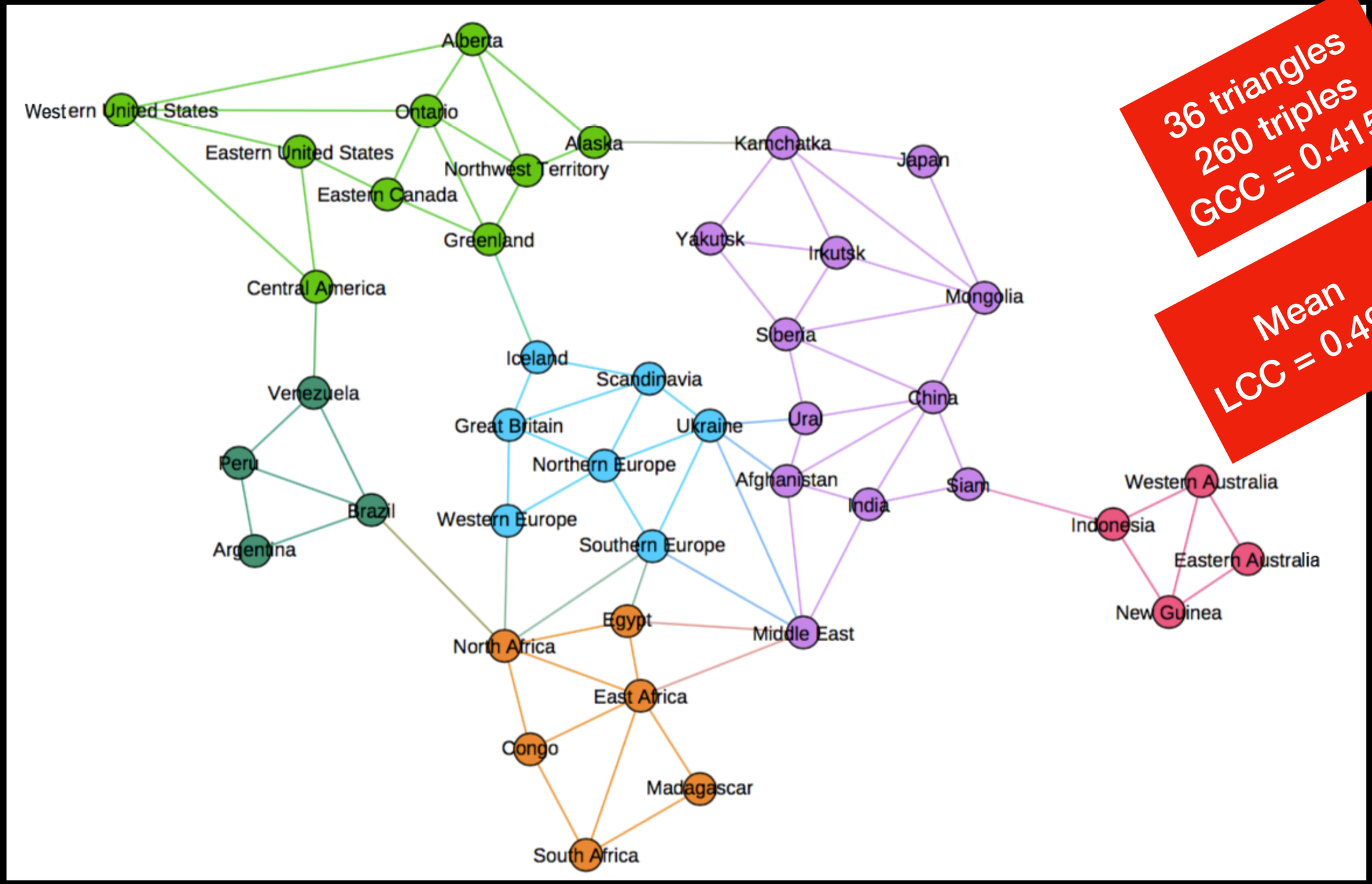
?? triangles
?? triples



Global Clustering Coef:

$$\frac{3 \times \text{triangles}}{\text{number of triples}} = \frac{3 \times 2}{8} = 0.75$$

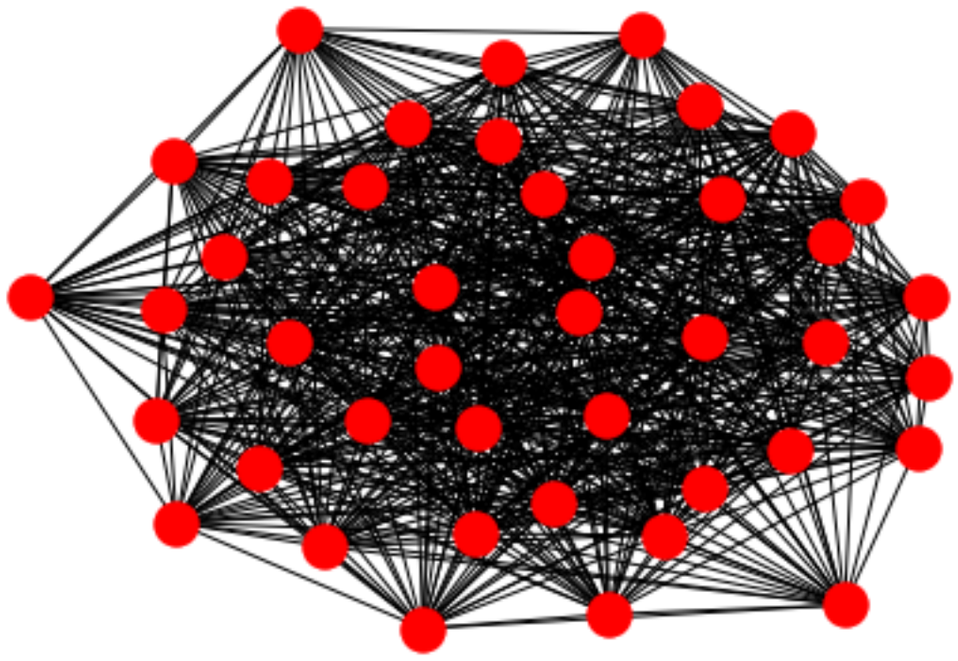
Another Cluster Metric: Global Clustering Coefficient

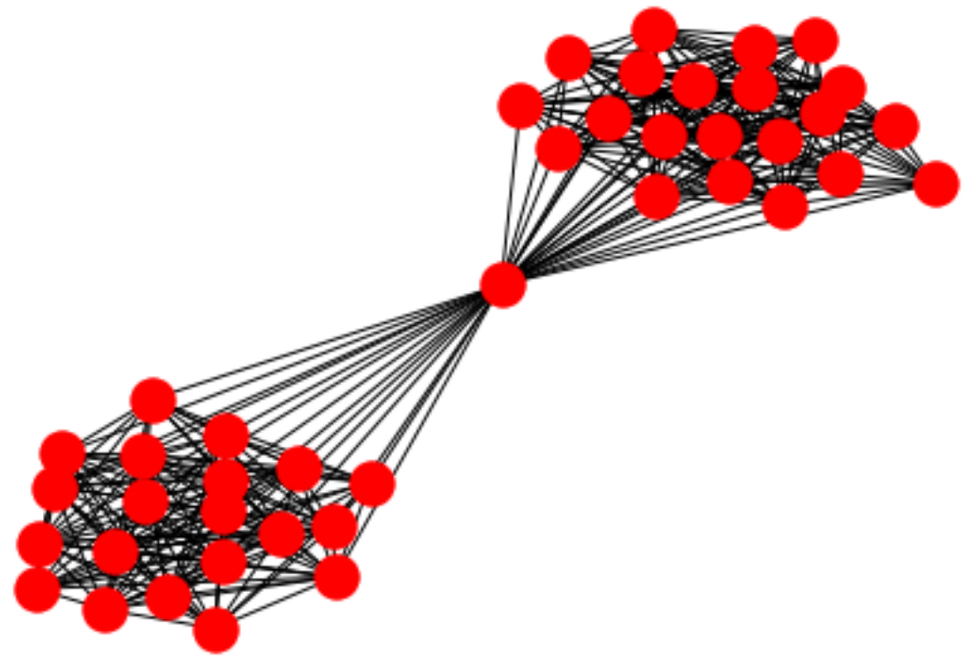


36 triangles
260 triples
GCC = 0.4154

Mean
LCC = 0.4921

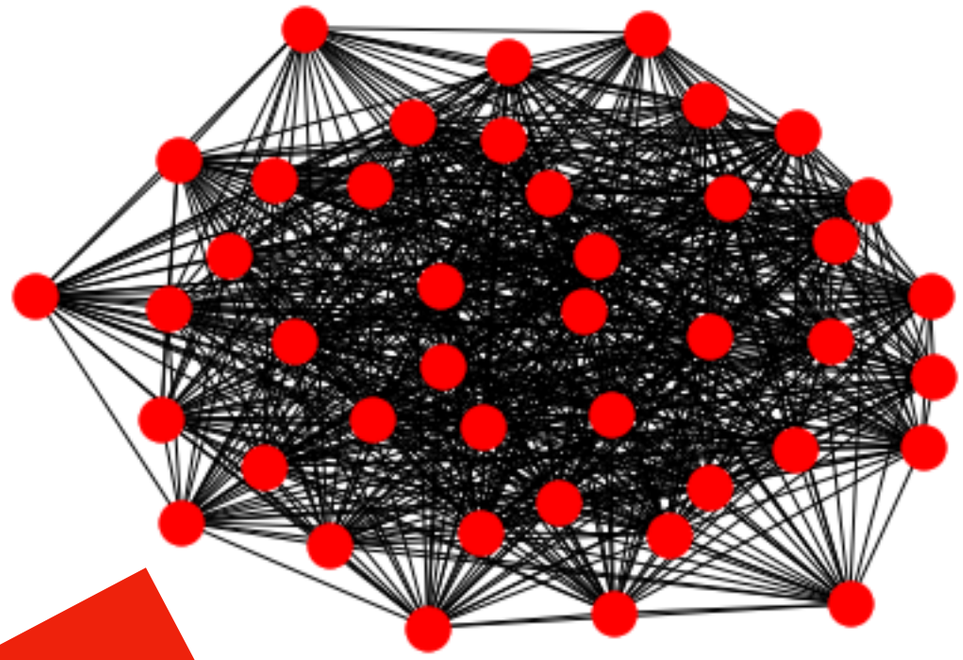
How **Centralized** are These Graphs?





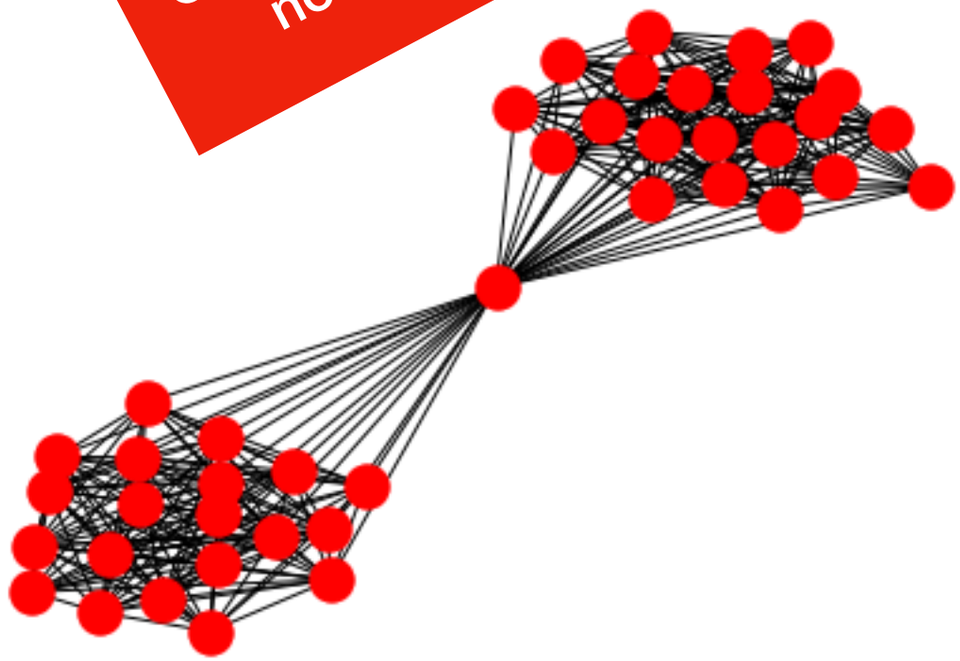
Concentration of Power

How Centralized are These Graphs?

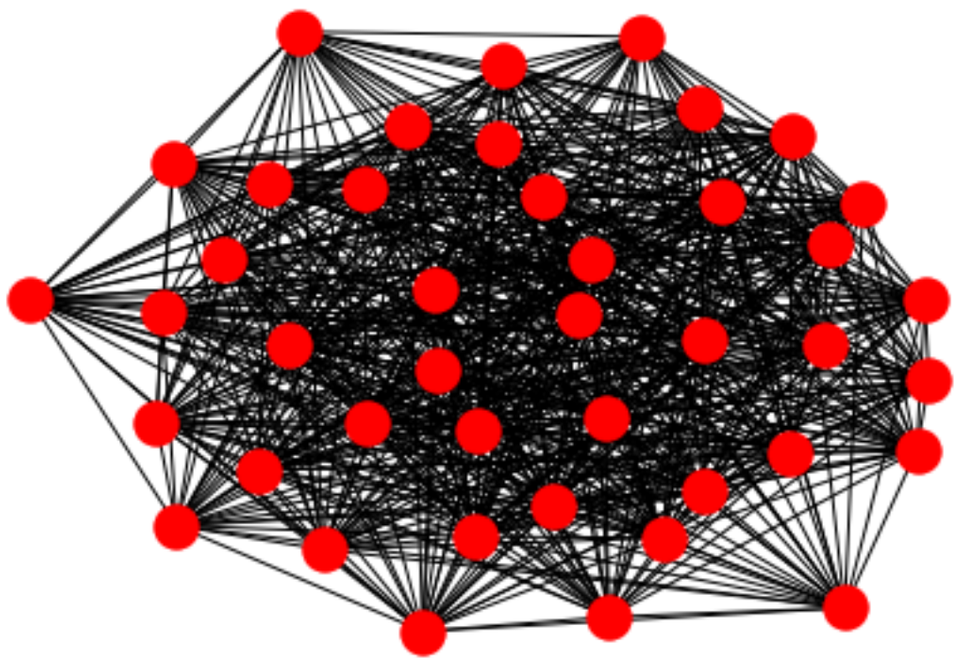


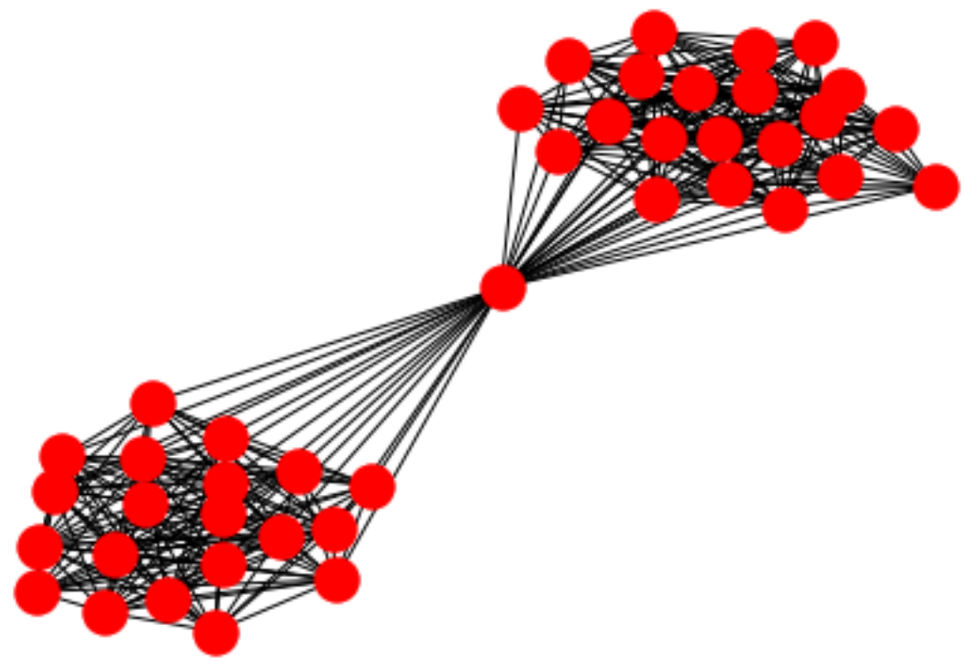
Pretty Uniform

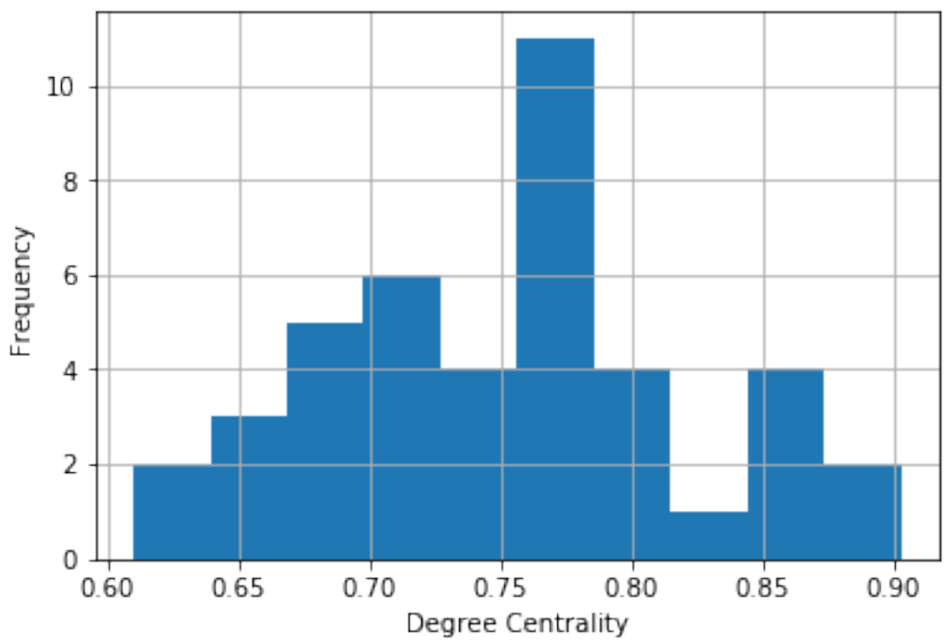
One central node

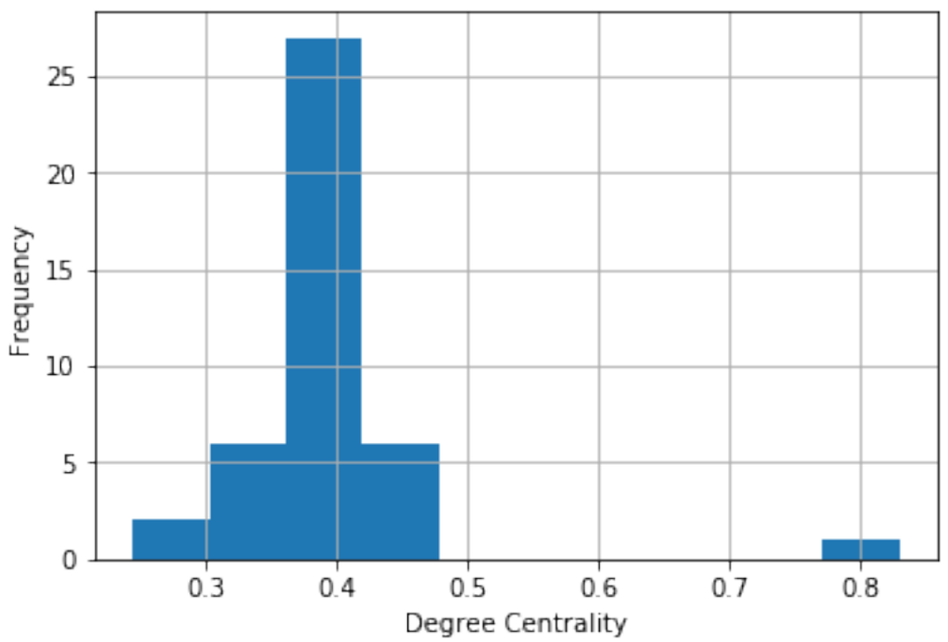


How **Centralized** are These Graphs?









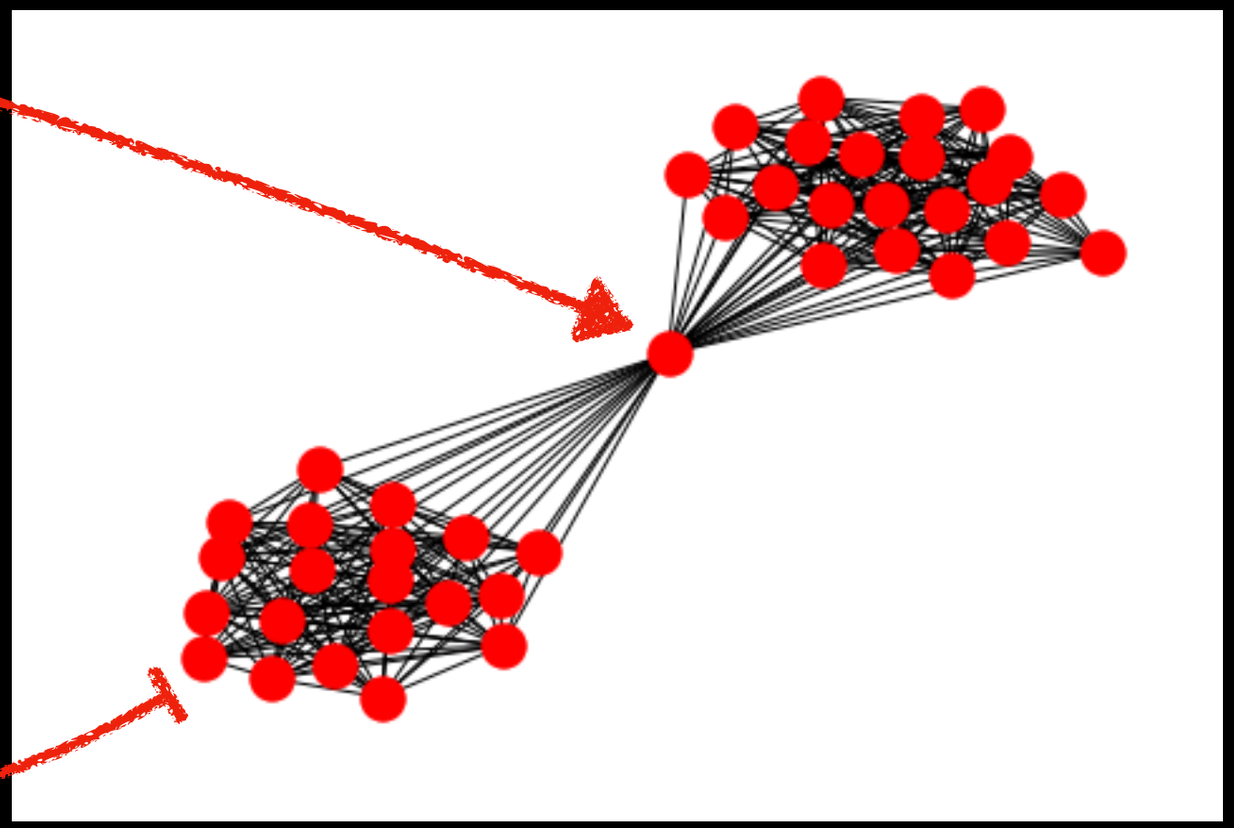
How Centralized are These Graphs?

Centralization Metric:

$$c^* = \max_i \{\text{centrality}(c_i)\}$$

$$S = \sum_i c^* - \text{centrality}(c_i)$$

$$S = 18.29$$



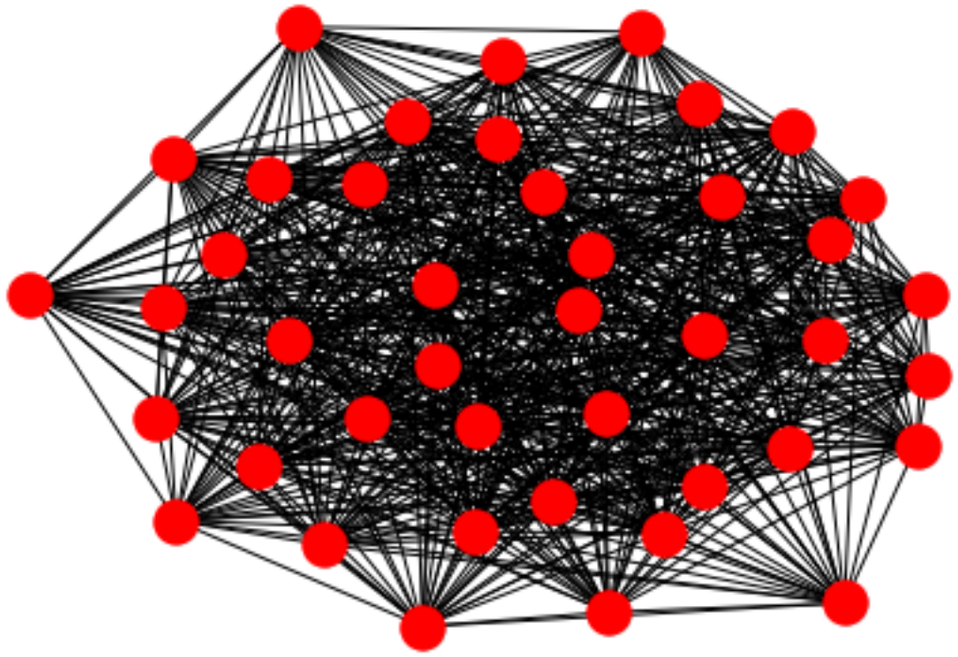
How Centralized are These Graphs?

Centralization Metric:

$$c^* = \max_i \{\text{centrality}(c_i)\}$$

$$S = \sum_i c^* - \text{centrality}(c_i)$$

$$S = 5.854$$



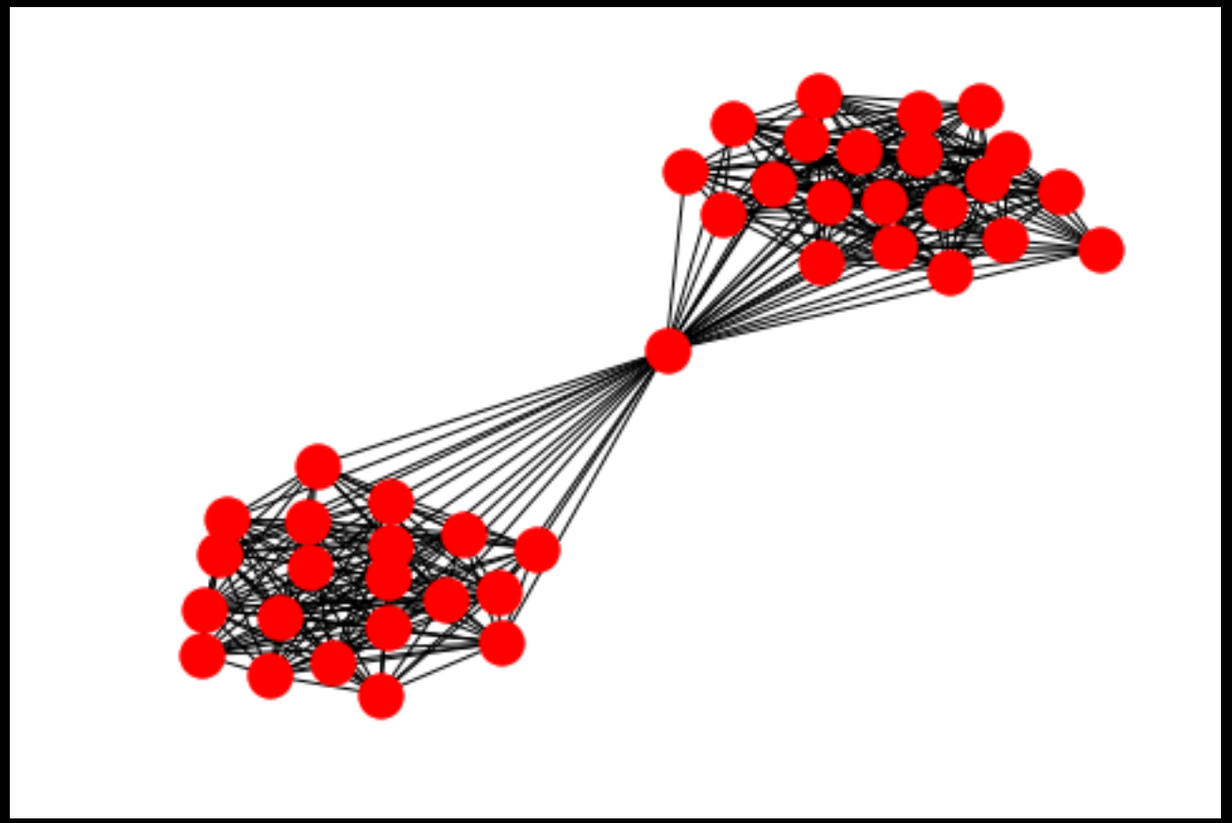
How Centralized are These Graphs?

Normalized Centralization Metric:

$$c^* = \max_i \{\text{centrality}(c_i)\}$$

$$\bar{S} = \frac{\sum_i c^* - \text{centrality}(c_i)}{\max S_n}$$

Maximum centralization score over all graphs of the same size

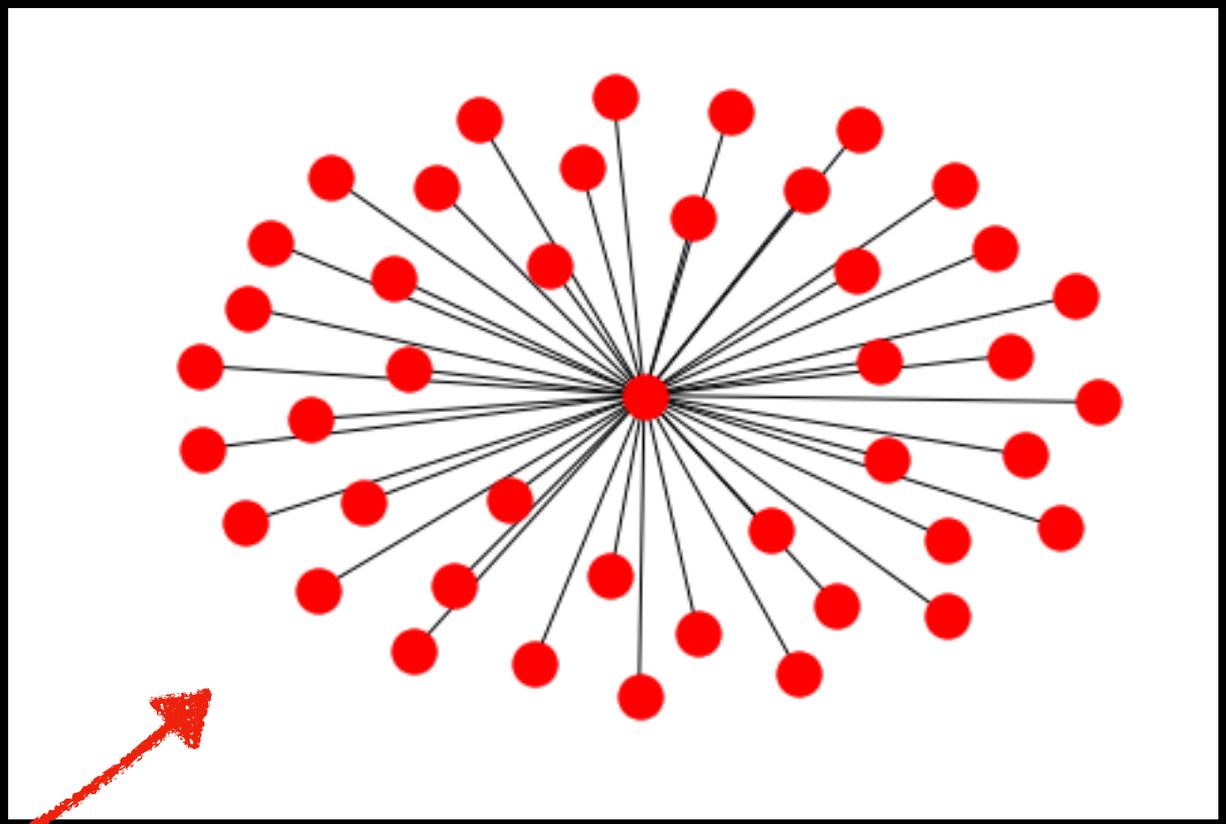


How <u>Centralized</u> are These Graphs?

<u>Normalized</u> Centralization Metric:

$$\max S_n ?????????$$

$$\max S_n \approx 41.00$$



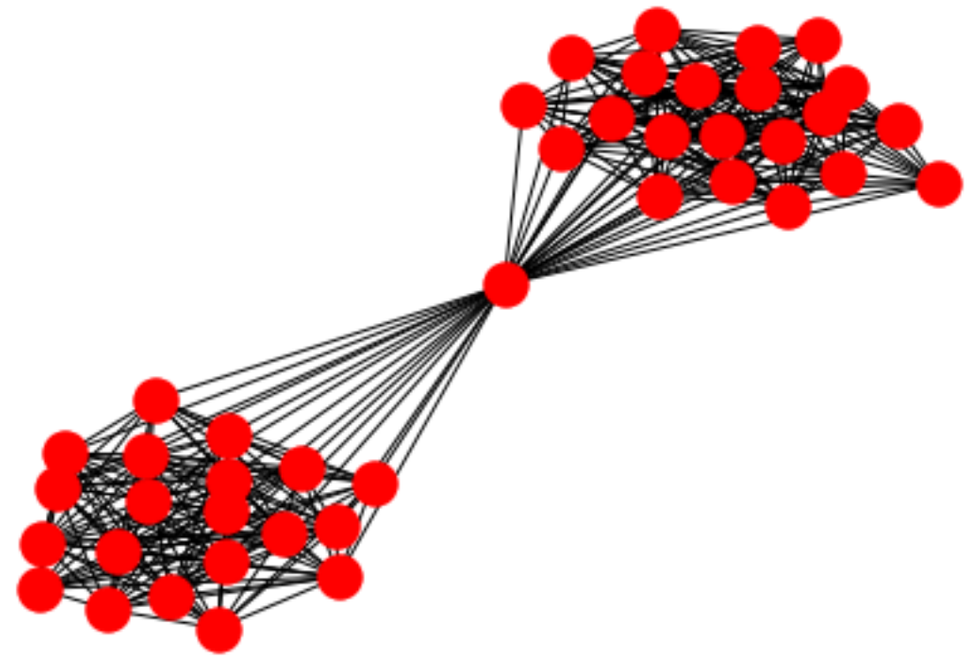
How Centralized are These Graphs?

Normalized Centralization Metric:

$$c^* = \max_i \{\text{centrality}(c_i)\}$$

$$\bar{S} = \frac{\sum_i c^* - \text{centrality}(c_i)}{\max S_n}$$

$$\bar{S} = \frac{18.29}{41.00} \approx 0.4461$$



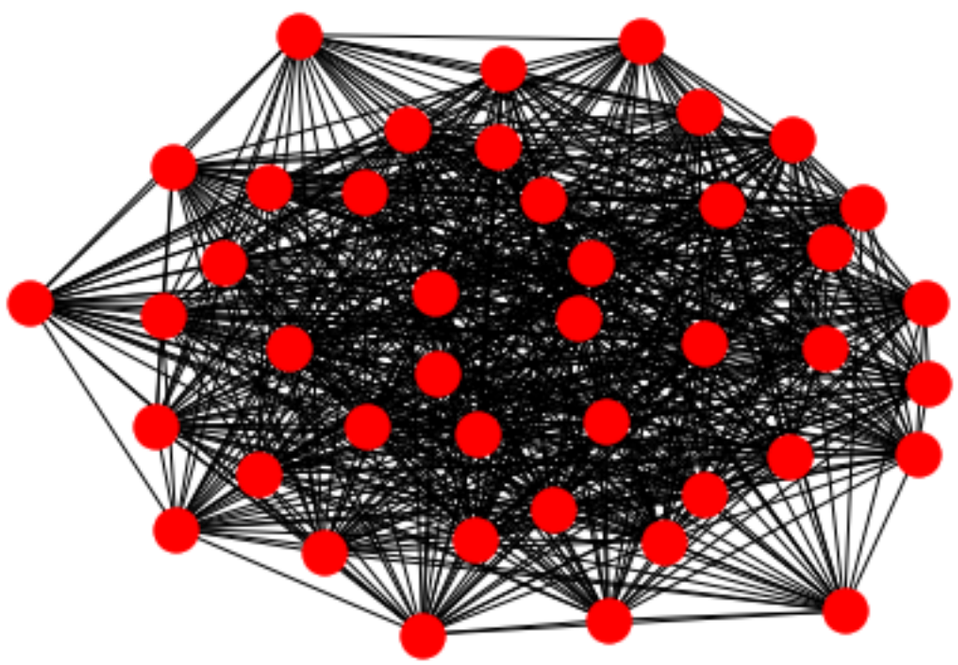
How Centralized are These Graphs?

Normalized Centralization Metric:

$$c^* = \max_i \{\text{centrality}(c_i)\}$$

$$\bar{S} = \frac{\sum_i c^* - \text{centrality}(c_i)}{\max S_n}$$

$$\bar{S} = \frac{18.29}{41.00} \approx 0.1428$$



How Centralized are These Graphs?

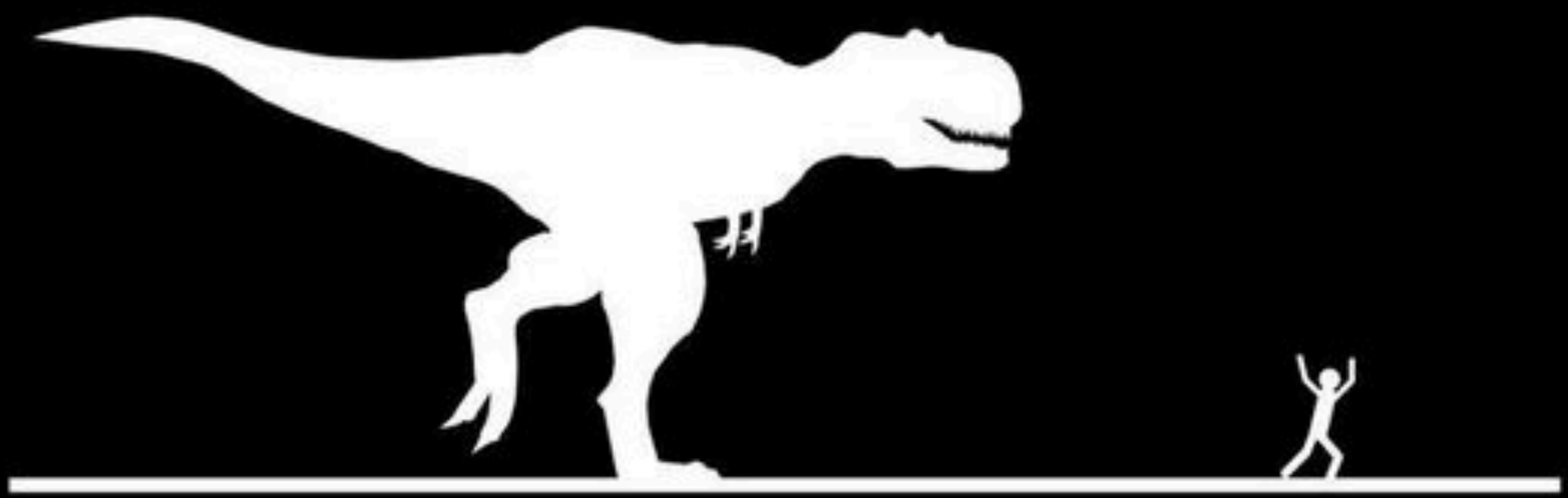
Normalized Centralization Metric:

$$c^* = \max_i \{\text{centrality}(c_i)\}$$

$$\bar{S} = \frac{\sum_i c^* - \text{centrality}(c_i)}{\max S_n}$$

Can be any centrality metric

HW2 posted



EXERCISE

Some motivation required.